
3.1 Uniform Circular Motion

With everything we have studied so far (Kinematics, Dynamics, Gravity), objects have always been moving in a straight line. Now we are talking about circles.

Uniform Circular Motion - constant circular movement
Eg. hammer throw, bucket of water on a string, fair rides, etc.


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Let's analyze the bucket of water on a string. As long as SPEED stays the same we can say the object has uniform circular motion.


If we look at the velocity at different points in the path of the bucket, we will see the magnitude of velocity remains the same but direction changes.


The instantaneous velocity (velocity of the bucket at any given point in time) is always perpendicular to the radius of the circle and tangent to the circle.

So if the magnitude of velocity remains constant, we have:

$$
\vec{v}_{\mathrm{ave}}=\frac{\Delta \vec{d}}{\Delta t}
$$

However, time means something different with circular motion. We use $T$ to represent the time it takes to complete one revolution around the circle(called the period).

Now, distance will also mean something different when we go around a circle. Distance around a circle is circumference, or $2 \pi r$.

If we make all the substitutions to the formula to take into account the fact that we are moving around a circle, we get the formula for velocity of an object moving in a circle:

$$
\left|\vec{v}_{\mathrm{c}}\right|=\frac{2 \pi r}{T}
$$

*formula found in Kinematics sections of formula sheet

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Ex.) If the water bucket has a period of 1.5 s and the string is 1.25 m long, what is the magnitude of the buckets velocity?


Ex.) A super-plane is flying at a height of 10000 m above sea level in a circular path around the planet (assume it can hold enough fuel to do this in one trip). If the velocity of the super-plane is $885 \mathrm{~km} / \mathrm{h}$, how long does it take the superplane to go around the world?


We've looked at velocity around a circle, now let's take a look at acceleration:

$$
\left|\vec{a}_{c}\right|=\frac{v^{2}}{r}
$$

Acceleration around a circle is called centripetal acceleration(meaning centre seeking) and is directed toward the centre of the circle.


Two "forces" that are often confused in the Physics world are centripetal and centrifugal. Below are the differences:

|  | Centrifugal Force | Centripetal Force |
| :---: | :---: | :---: |
| Meaning | Tendency of an object following a curved path to fly away from the center of curvature. Might be described as "lack of centripetal force." | The force that keeps an object moving with a uniform speed along a circular path. |
| Direction | Along the radius of the circle, from the center towards the object. | Along the radius of the circle, from the object towards the center. |
| Example | Mud flying off a tire; children pushed out on a roundabout. | Satellite orbiting a planet |
| Formula | $\mathrm{Fc}=\mathrm{mv} 2 / \mathrm{r}$ | $\mathrm{Fc}=\mathrm{mv} 2 / \mathrm{r}$ |
| Defined by | Chistiaan Hygens in 1659 | Isaac Newton in 1684 |
| Is it a real force? | No; centrifugal force is the inertia of motion. | Yes; centripetal force keeps the object from "flying out". |

Important for this course: centripetal acceleration always points towards the centre of the circle.

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Ex.) A car takes a curve of radius 15 m at $45 \mathrm{~km} / \mathrm{h}$. What is the car's acceleration?

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Newton's Second Law tells us that where there is acceleration, there is a force in the same direction.

If we have centripetal acceleration. it follows that we would have centripetal force:

$$
F_{c}=\frac{m v^{2}}{r}=\frac{4 \pi^{2} r m}{T^{2}}
$$

This centripetal force can be friction, tension, gravity...whatever.

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Read: Pg. 242-256.

## Crash Course Physics

https://www.youtube.com/watch?v=bpFK2VCRHUs
Khan Academy
https://www.youtube.com/watch?v=FfNgm-w9Krw


We have always drawn horizontal circular motion like this:


But it really should have been drawn like this:


Because now we look at vertical circular motion like this: ( $\overrightarrow{F g}_{\text {g }}$ is now at play)




Ex.) Neglecting friction, what is the minimum speed a Hot Wheels car must go around a vertical loop of radius 15.0 cm to keep from falling off?


Ex.) What is the maximum radius a roller coaster loop can be if a cart with speed of $20.0 \mathrm{~m} / \mathrm{s}$ is to go around safely?


Ex.) What is the force the roller coaster track is providing to a 102 kg cart traveling at $15.0 \mathrm{~m} / \mathrm{s}$ around a 7.0 m radius loop?



It would seem like the force acting on the water must be acting outward to keep the water in the bucket, but it is not. The centripetal force is pointed towards the centre of the circle. And this force keeps things in a circular pattern, not a pattern where the water will fall.

Some people will refer to the "force" keeping the water in the bucket as "centrifugal force." But we know this isn't a real force. Physicists will refer to these "made up" forces as phantom forces or fictitious forces.


Ex.) A string can hold a force of 135 N before breaking. If a 2.00 kg object is tied to the end of this string $(\mathrm{L}=1.10 \mathrm{~m})$, how fast can I spin it vertically before the string breaks?

vertical circle with $r=0.75 \mathrm{~m}$. What is the 0 of the swing to maintain uniform circular


Satellite - any object that orbits around a central object (usually a planet)

Artificial Satellites - communications satellites, Space Shuttle, Hubble, ISS, space junk

Natural Satellites - moons of planets, planets orbiting the Sun

Geosynchronous Satellite - a satellite that orbits Earth once per day


For the projectile problems we worked with there was an unstated assumption that the Earth was flat. However, we know that the Earth is in fact spherical, although not perfectly so. With this in mind, Sir Isaac Newton reasoned that some strange things would happen if one could horizontally project an object at high speeds.

At low speeds, a horizontal projectile will fall toward and hit the ground in a short time. As the speed of the horizontal projectile is increased, it will land further and further away from the starting point. For a flat Earth the projectile would always hit the ground; no matter how fast the projectile went, gravity would pull it down to the ground.


However, since the Earth is round, the curvature of the Earth affects where the projectile lands. As the diagram indicates, the greater the horizontal speed of the projectile, the more the Earth's curvature comes into play. Eventually, a critical speed is reached where, even though the projectile is in constant freefall, it would not hit the Earth, rather, it would become a satellite in orbit around the Earth.

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Newton was able to calculate the speed required to put an object into an orbit which would just skim the surface of a smooth Earth. Newton realized that a satellite orbiting the Sun, the Earth, or some other body is simply a case of uniform circular motion. He reasoned that the gravitational force would act as the centripetal force for a circular orbit.

$$
\begin{aligned}
& \mathrm{F}_{\mathrm{c}}=\mathrm{F}_{\mathrm{g}} \\
& \frac{\mathrm{mv}}{\mathrm{r}}=\frac{\mathrm{GMm}}{\mathrm{r}^{2}} \\
& \mathrm{v}^{2}=\frac{\mathrm{GM}}{\mathrm{r}} \\
& \mathrm{v}=\sqrt{\frac{\mathrm{GM}}{\mathrm{r}}} \quad \begin{array}{l}
\text { (the mass of the satellite cancels, and one } \mathrm{r} \text { cancels as well) } \\
\text { where: } \\
\text { M mass of body being orbited } \\
\text { r distance from the center of } M \text { to the satellite }
\end{array}
\end{aligned}
$$

## Note

1. The orbit does not depend on the mass of the satellite as long as the mass of the planet or star around which the satellite is orbiting is much, much larger than the mass of the satellite.
2. The treatment of orbits that we shall work with is quite limited. For example, to calculate the escape velocities of satellites requires that we account for gravitational potential energy. In addition, we will limit ourselves to circular and not elliptical orbits.


Ex.) What is the speed of orbit for a satellite orbiting Saturn if the radius of orbit is $6.43 \times 10^{7} \mathrm{~m}$ ?


Recall that the derivation of the formula we used came from the idea that the force of gravity is supplying the centripetal force. We have more choices for these equations depending on the unknown in the problem:

$$
\begin{gathered}
\vec{F}_{g}=m g \\
\vec{F}_{g}=\frac{G m_{1} m_{2}}{r^{2}}
\end{gathered} \quad F_{c}=\frac{m v^{2}}{r}=\frac{4 \pi^{2} r m}{T^{2}}
$$

In the following examples you need to choose the appropriate formulas to use based on the variables given and the unknown.


Ex.) Galileo discovered 4 moons of Jupiter, listed below. Also listed are their periods of revolution and their orbital radii (centre to centre). From this data, determine the mass of Jupiter.

| moon | period (days) | distance $\left(10^{6} \mathrm{~m}\right)$ |
| :---: | :---: | :---: |
| lo | 1.769137786 | 422 |
| Europa | 3.551181041 | 671 |
| Ganymede | 7.154552960 | 1070 |
| Callisto | 16.68901840 | 1883 |

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Ex.) What is the speed of the moon lo based on the information from the previous slide?


Ex.) The average distance from the centre of the Earth to the centre of the Moon is $3.85 \times 10^{8} \mathrm{~m}$ ? What is the period of orbit of the Moon around the Earth?


Ex.) Determine the height from the surface of the Earth of a geo-sync satellite.

Read: Pg. 276-286
Questions: Pg. 286 \# 9-13.

3.4 Kepler's Laws of Planetary Motion

An astronomical observer named Tycho Brahe (1546-1601) spent more than 20 years making careful measurements and observations of the heavens. His measurements were accurate to 1/1000th of a
degree. Johannes Kepler, an assistant to Brahe, wanted to use Brahe's data to plot the orbit of Mars. For years Brahe kept promising the data to Kepler. Finally, Kepler stole Brahe's data after Brahe died in 1601. Kepler spent 16 years working on and plotting the orbit of in 1601. Kepler spent 16 years working on and plotting the orbit of
Mars, producing 900 pages of calculations. Fortunately for Kepler,


Mars, producing 900 pages of calculations. Fortunately for Kepler,
Mars' orbit is just enough of an ellipse that Brahe's data could not be Mars' orbit is just enough of an ellipse that Brahe's data could not be
forced to conform to a circular orbit. Kepler discovered three laws of planetary motion:
planetary motion:
 from the sun to the planet that the line ceses which ace proportional to th $A B$ is the cume an that for $B C, C D$, and so on.

1. Planets orbit the sun in elliptical orbits with the sun at one focus of the ellipse.
2. A straight line joining the sun to a planet sweeps out equal areas in equal times.
3. The cube of a planet's mean distance $(r)$ from the sun is proportional to the square of the period of revolution $(T)$ of a planet.

$$
\frac{\mathrm{r}^{3}}{\mathrm{~T}^{2}}=\mathrm{k}
$$

## Random Scientist Facts

Kepler's mother was accused of witchcraft.

Kepler devised the eyeglass for nearsightedness and farsightedness

- Tycho Brahe liked to drink and get in fights...his nose was cut off so he had a bronze nose made and obsessively rubbed it with oil. He had a gold nose for special occasions.


Kepler gave no explanation of why planets go around the sun. His laws are only descriptive. However, Sir Isaac Newton provided the explanation for why the moon, Earth, Sun, planets and stars moved and behaved as they did. Newton was able to explain Kepler's three laws of planetary motion. He reasoned that if we make the approximation that a planet's orbit is circular, than the gravitational attraction between the planet and the Sun provides the centripetal force to maintain the planet's orbit around the Sun.

$$
\begin{aligned}
& \mathrm{F}_{\mathrm{c}}=\mathrm{F}_{\mathrm{g}} \\
& \frac{4 \pi^{2} \mathrm{mr}}{\mathrm{~T}^{2}}=\mathrm{G} \frac{\mathrm{Mm}}{\mathrm{r}^{2}} \\
& \frac{\mathrm{r}^{3}}{\mathrm{~T}^{2}}=\frac{\mathrm{GM}}{4 \pi^{2}} \\
& \text { or } \\
& \frac{r^{3}}{\mathrm{~T}^{2}}=\mathrm{k} \quad \text { where } \mathrm{k}=\frac{\mathrm{GM}}{4 \pi^{2}}=3.35 \times 10^{18} \mathrm{~m}^{3} / \mathrm{s}^{2} \text { for the Sun }
\end{aligned}
$$

This is Kepler's third law.

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Kepler's First Law: planets move in ellipses, with the Sun at one focus

Kepler's Second Law: planets sweep out equal areas in equal times (ie. their orbital speed is not fixed)

Kepler's Third Law: the square of the period of a plane
 orbit divided by the cube of its orbital radius is a constant

$$
\frac{\mathrm{r}^{3}}{\mathrm{~T}^{2}}=\mathrm{k}
$$




Kepler's Third Law was "proven" with data that Tycho Brahe had collected. Years later, Newton determined that gravity keeps planets in their orbits and that centripetal force is supplied by gravity proving Kepler's Third Law.


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Work is a change in energy...but this is saying a lot more than it seems.
From the previous slide, we can deduce that: work is proportional to mass

## $\mathbf{W} \propto \mathbf{m}$

work is proportional to displacement
$\mathbf{W} \alpha \overline{\mathbf{d}}$
work is proportional to acceleration (ie. gravity)

$$
\mathbf{W} \propto \overrightarrow{\mathbf{a}}
$$

So...

$$
W=\vec{F} \cdot \bar{d}
$$

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Work is scalar (only a magnitude). However, force and displacement must be in the same direction for work to have been done!!

A. Figure 6.6


A Figure 6.7

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Ex.) How much work is done in lifting a 25 kg box to a height of 5.0 m ?

Ex.) How much work is needed, after lifting the box to carry it horizontally 250 m ?


What about this fella...there are two dimensions to the force he is applying to the box so is work being done?


A Figure 6.5 When a force acts on an object, resulting in a displacement, only the component of the force that acts parallel to the displacement does work. If the box moves
horizontally, only the horizontal component, $\bar{F}_{\text {l }}$, does work.

The answer is yes...sort of. When force is being applied at an angle, $\theta$, we can break the force down into parallel and perpendicular components. Therefore, if one component of the force is acting in the same direction as the displacement, some work is being done. We can find it using our old pal, Trigonometry.

$$
W=|\vec{F}||\vec{d}| \cos \theta
$$

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Ex.) Ryan is shoveling the walk. A force of 150 N is applied down the shovel handle, which makes an angle of $35.0^{\circ}$ with the horizontal. Ryan pushes the shovel 10.0 m . How much work is being done on the shovel?
**units for work are Joules


Ex.) A 7.00 kg crate is pushed up a hill with an incline of $11.0^{\circ}$ for 3.00 m . A 90.0 N horizontal force, parallel to the ground, is applied to the crate. The coefficient due to friction is 0.200 .
a) How much work is done?
b) How much work is done by gravity against the crate?
c) How much work is done by friction against the crate?


Read: Pg. 293-294.
Questions: Pg. 294 Practice Problems.


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Ex.) A 70 kg person climbed a 12 m ladder. Calculate the potential energy with respect to:
a) The ground.
b) The roof ( 11 m above the ground).
c) A tree, 7.0 m below the top of the ladder.


Ex.) A pendulum bob of mass of 2.00 kg is fixed from the ceiling by a string of length 1.00 m . If the bob is pulled 0.750 m to one side, what is its potential energy with respect to its equilibrium position?



Ex.) A 10.0 N ball is accelerated uniformly from rest at a rate of $2.50 \mathrm{~m} / \mathrm{s}^{2}$. What is the kinetic energy of this object after it has accelerated a distance of 15.0 m ?


Ex.) An 8.0 kg rock is dropped from a height of 7.0 m . What is the kinetic energy of the rock as it hits the ground?


Ex.) By what factor must the kinetic energy increase to cause the speed to triple?


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Note: You will notice there are two very similar formulas on your formula sheet for Hooke's Law. One with a negative and one without. This is because as you stretch a spring, you can decided which direction you want to be positive or negative and adapt the formula accordingly.

If you find the area under a weight vs. stretch graph, you are finding work (and therefore energy).

$$
\begin{aligned}
& \begin{array}{l}
\mathrm{A}=1 / 2 \mathrm{bh} \\
\\
\text { or } \\
\mathrm{E}=1 / 2 \mathrm{Fx}
\end{array} \\
& \mathrm{E}=1 / 2(\mathrm{kx}) \mathrm{x} \\
& \text { which yields: } \\
& E_{s}=\frac{1}{2} k x^{2} \quad \begin{array}{l}
\mathrm{E}_{\mathrm{s}}=\text { elastic potential energy (J) } \\
\mathrm{k}=\text { spring constant }(\mathrm{N} / \mathrm{m}) \\
\mathrm{x}=\text { displacement of spring (m) }
\end{array}
\end{aligned}
$$



Ex.) The following graph shows how a force causes change in position as it stretches a spring.
a) Calculate the energy stored in the spring when the force is 36.0 N .
b) Compare the energy when the force is 46.0 N to the energy stored in the spring when the position is 7.00 cm .


Questions: Pg. 305 \# 4, 8, 11.

3.8 Mechanical Energy

Mechanical Energy - the sum of all energies acting on a given system (ie. a skydiver has potential and kinetic energies at any given time in the air)

Since mechanical energy is the sum of all energies and work is the change in energy, we could say:

$$
\begin{aligned}
& \text { The Work-Energy Theorem } \\
& \qquad \mathbf{W}=\Delta E_{k}+\Delta E_{p}
\end{aligned}
$$

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Ex.) A farmer is hauling feed and he lifts a 9.00 kg bucket up 5.00 m of rope.
This takes a force of 150 N upwards.
a) What work does the farmer do on the feed?
b) What is the change in PE of the feed?
c) What is the change in KE of the feed?


Ex.) A 150 kg sled and rider are pushed up a hill with a vertical height of 6.53 m . The initial velocity of the rider is $2.50 \mathrm{~m} / \mathrm{s}$ and the final velocity of the rider is $5.80 \mathrm{~m} / \mathrm{s}$.What amount of work is needed to push the sled up the hill?


Ex.) A 450 kg care package for soldiers is dropped from an airplane and reaches a velocity of $35 \mathrm{~m} / \mathrm{s}$ at 350 m . What is the mechanical energy of the package? (Hint: pick appropriate units of energy)


Why is mechanical energy important? Because of the big idea we have studied throughout this course...THE LAW OF CONSERVATION OF ENERGY!

In an isolated system, mechanical energy is conserved. Energy is not created or destroyed, only changed in form.

Isolated System - a system in which energy cannot enter or leave
Energy is Conserved - the total amount of energy is constant but may be in constant flux


Ex.) A frictionless roller coaster car has a mass of 200 kg and travels along a path as shown:
Calculate the :
a) PE at the first hill
b) KE and speed at the bottom of the dip

c) speed at the top of the second hill

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Ex.) Draw (on the same set of axes) an $E_{p}$ vs. time graph and a $E_{k}$ vs. time graph for an ideal pendulum swinging back and forth. Consider the starting point to be when the bob is pulled back to the side then released.


Ex.) Use two different methods to calculate the speed an object would hit the ground with if dropped from 12.0 m.


Ex.) A pendulum is dropped from the position shown 0.25 m above equilibrium. What is the speed of the bob as it passes through the equilibrium position?



Ex.) A roller coaster traveling on a frictionless track is shown. If the speed of the car at $A$ is $3.0 \mathrm{~m} / \mathrm{s}$, what is the speed at $B$ ?



Questions: Pg. 310 \# 6, 7, 8.
Pg. 315-316 \# 1, 3, 4.
Read: Pg. 319-322.


We know work is the change in energy but what is power?
Power - the rate of change of work
Recall from Math 10C that "rate of change" means slope.
Another way to describe power is "the amount of energy per second applied."

$$
P=\frac{\Delta E}{\Delta t}=\frac{W}{\Delta t}
$$

$P=\operatorname{power}(\mathrm{J} / \mathrm{s}=\operatorname{Watt}(\mathrm{W}))$
W = work ( J )
$E=$ energy $(J)$
$\mathrm{t}=$ time $(\mathrm{s})$

Power was traditionally measure with the unit: horsepower (I'm sure you can imagine why) but this was cumbersome and thus replaced with the Watt.

$$
1 \mathrm{hp}=746 \mathrm{~W}
$$



Note that when you are given the Power formula; $\mathrm{P}=\mathrm{W} / \Delta \mathrm{t}$, you can derive different formulas not on the formulas sheet. This is because we have a couple formulas for work; $\mathrm{W}=\mathrm{Fd}, \mathrm{W}=$ mad. One useful derivation is shown:

$\mathbf{P}=\underline{\mathbf{F}} \mathbf{d}$
$\Delta \mathbf{t}$
$\mathbf{P}=\overrightarrow{\mathbf{F}} \stackrel{\rightharpoonup}{\mathbf{v}}$

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Ex.) You lift a 25.0 kg box to your waist $(0.800 \mathrm{~m})$ in 1.20 s . What is your power output?

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Ex.) A plane's engine exerts a thrust of $1.20 \times 10^{4} \mathrm{~N}$ to maintain a speed of 450 $\mathrm{km} / \mathrm{h}$. What power is the engine generating?

