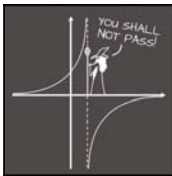
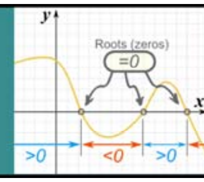


Unit 3: Polynomial, Radical, and Rational Functions



3.1 Characteristics of Polynomials

Types of Polynomial Functions(the variable has a whole number exponent):

Constant:

Linear:

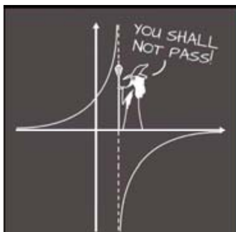
Quadratic:

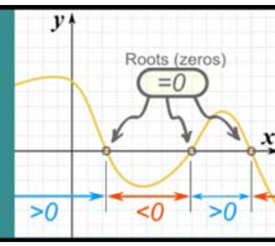
Cubic:

Quartic:

Quintic:

Oct 18-8:30 AM





Characteristics:

Degree:

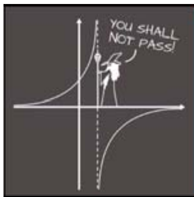
Leading Coefficient:

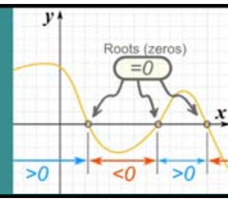
Constant Term:

X-Intercepts:

End Behaviour:

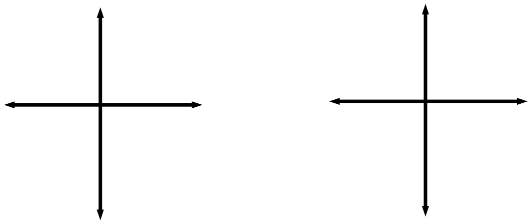
Oct 18-8:39 AM



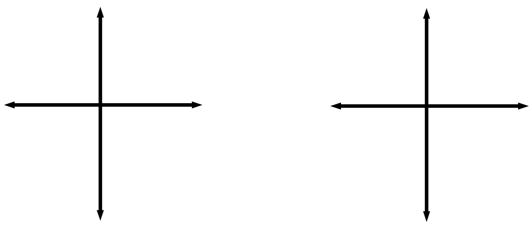


Investigate the End-Behaviour of Even Functions: x^2 and x^4

$f(x) = x^2$



$f(x) = x^4$



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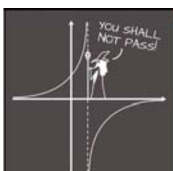
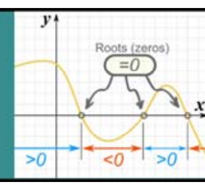
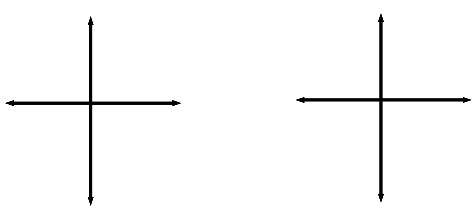


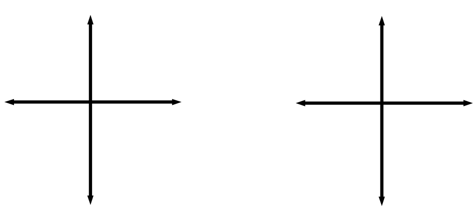


Investigate the End-Behaviour of Odd Functions: x^3 and x^5

$f(x) = x^3$



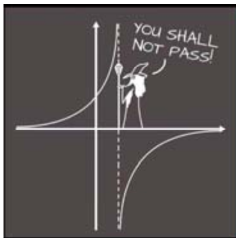
$f(x) = x^5$



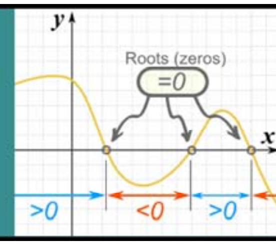
Worksheet.

Pg. 114 #1, 2, 4, 6, 7, 9.

Oct 18-8:48 AM



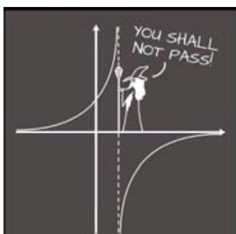
Unit 3: Polynomial, Radical, and Rational Functions

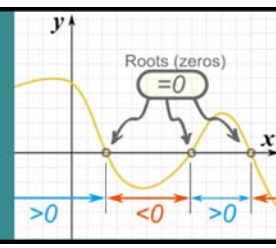


3.2 Synthetic Division and The Remainder Theorem

Recall: Long Division with Numbers

Oct 18-9:23 AM





Terminology:

Quotient and Remainder

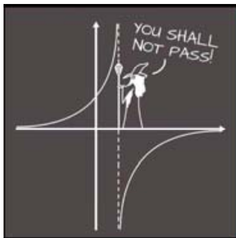
Divisor / Dividend

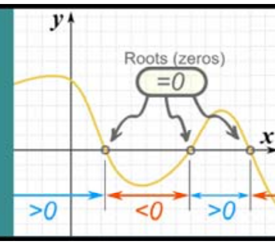
f(x) = Dividend
g(x) = Divisor
q(x) = Quotient and Remainder

then,

f(x) = g(x) q(x) = Divisor (Quotient + [Remainder / Divisor])

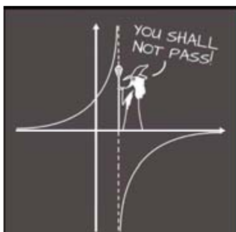
Oct 18-9:29 AM

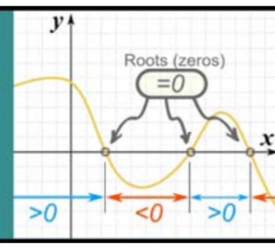




Now, long division with polynomials:

Oct 18-9:30 AM

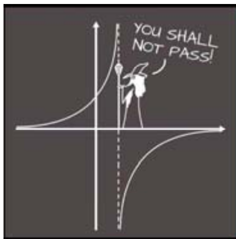


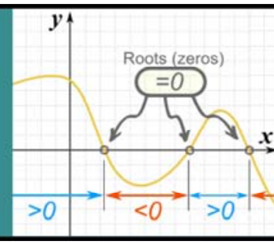


Long division is a great way to divide polynomials. However, there is a simpler way that removes all the variables and just works with the coefficients; Synthetic Division.

Synthetic Division:

Oct 18-9:31 AM



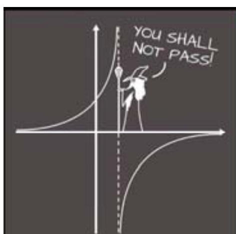


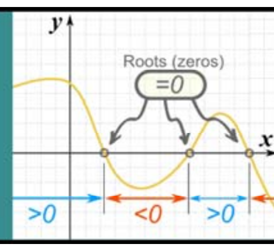
What does this tell us about polynomials?

In Math 20-1: $f(x) = x^2 + 7x + 6$ factors to $(x + 6)(x + 1)$ so x-int's are -6, -1.

In Math 30-1: We use synthetic division to see if there is a remainder. **If there is a remainder, the divisor is not a factor of the dividend.**

Oct 18-9:33 AM



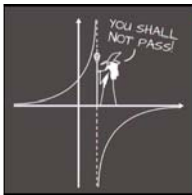


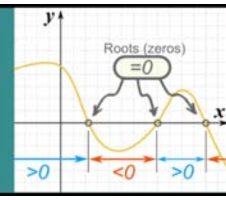
The Remainder Theorem:

> If a polynomial, f(x), is divided by (x - a), the remainder is the constant f(a), and
>
> dividend = quotient x divisor + remainder
>
> $$f(x) = q(x) \cdot (x - a) + f(a)$$
>
> where q(x) is a polynomials with degree one less that the degree of f(x).

So with our quadratic example:

Oct 18-9:40 AM



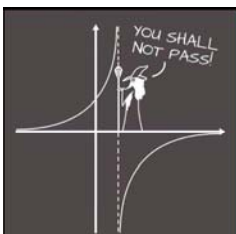


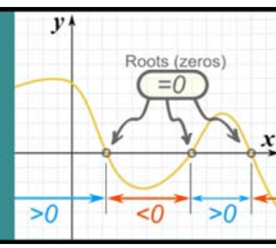
Ex.) Divide $2x^3 + 3x^2 - 4x + 15$ by $(x+3)$.

Ex.) Divide $5x^4 - 3x^2 + 2x - 7$ by $(x+2)$.

Worksheet

Oct 18-9:46 AM

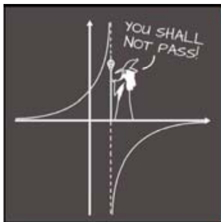


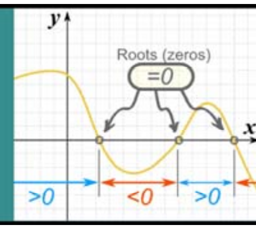


Ex.) Use the remainder theorem to find the remainder when $P(x) = x^3 - 10x + 6$ is divided by $x-4$.

Ex.) Use synthetic division to divide $4x^5 - 3x^3 + 7x^2 - 6$ by $(x+1)$.

Oct 18-9:49 AM

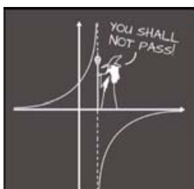




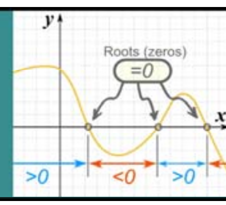
Ex.) When $P(x) = x^3 + 4x^2 - x + k$ is divided by $(x-1)$, the remainder is 3. What is the value of 'k'?

Pg. 124 # 3, 6, 8, 10.

Oct 18-9:53 AM



Unit 3: Polynomial, Radical, and Rational Functions



3.3 The Factor Theorem

The Factor Theorem states that if *f(a) = 0* for a polynomial, then *(x-a)* is a factor of the polynomial *f(x)*.

Ex.) Fully factor the following polynomials:

a) $P(x) = x^3 - x^2 - 5x + 2$

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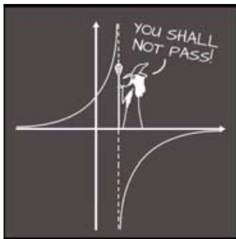

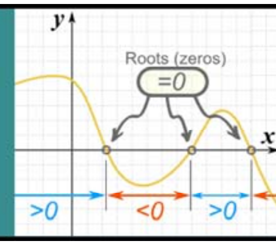

b) $P(x) = x^3 - x^2 - 4x + 4$

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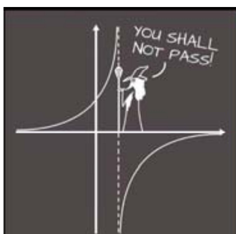

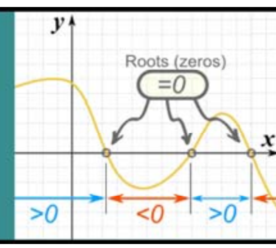

c) $P(x) = 2x^3 - 5x^2 - 4x + 3$

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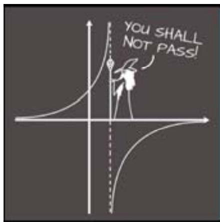

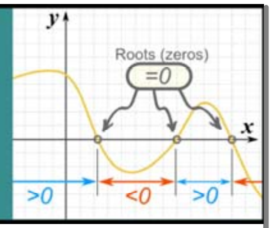

d) $P(x) = x^4 - 5x^3 + 2x^2 + 20x - 24$

Pg. 133 # 1, 3, 5, 6, 7, 11, 12.

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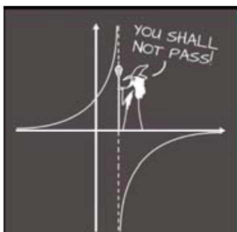
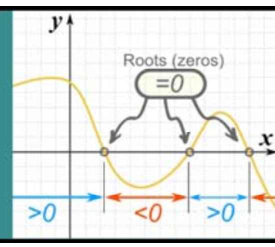


Unit 3: Polynomial, Radical, and Rational Functions



3.4 Graphing Polynomials

$$P(x) = a(x+b)(x-c)(x-d)(x-e)$$

Degree: 4

$a < 0$

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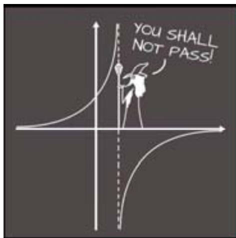

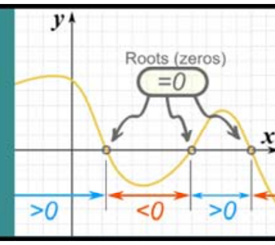

P(x) = a(x+b)(x-c)(x-d)

Degree: 3

a > 0

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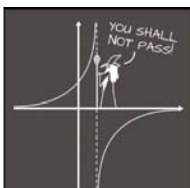
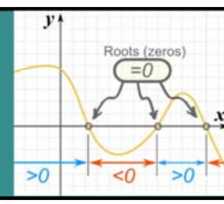




Multiplicity: - the power of the factor

Even Power: the graph is tangent to the x-axis

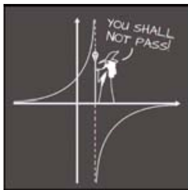
Odd Power: the graph cuts the x-axis

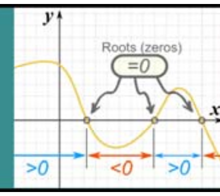
Ex.) Sketch and find degree, multiplicity, and the roots of the following:

a) $P(x) = a(x + 5)(x - 3)^2$

b) $P(x) = a(x + 3)^3(x - 4)^2$

Oct 19-1:18 PM





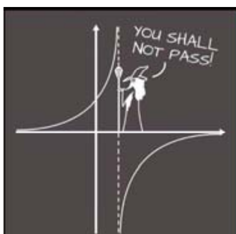
Ex.) Given $P(x) = 3(x - 2)(x + 1)(x - 4)^2$, sketch the graph.

Ex.) Sketch $P(x) = -2(x + 3)^2(x - 5)^3$.

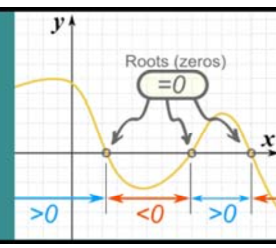
Pg. 147 # 2, 4, 5, 7ac, 10acd.

Worksheet.

Oct 19-1:25 PM



Unit 3: Polynomial, Radical, and Rational Functions



3.5 Determining the Equation of a Polynomial

$$P(x) = a(x - b)(x - c)(x - d)...$$

Here's the information you may be given in a question or on a graph in order to write the equation:

- roots (x-int, related to factors)
- multiplicities of roots
- degree of the equation
- a point on the graph *(x,y)*

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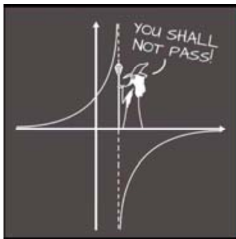

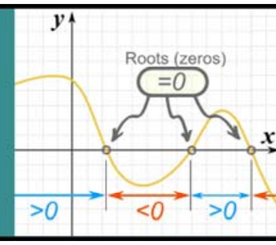

Ex.) A polynomial function has zeros at -3 and 4 and passes through the point (1,15). The multiplicity of (-3,0) is 1 and the multiplicity of (4, 0) is 2. Find *P(x)*.

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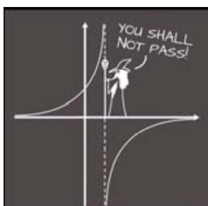
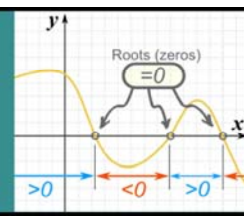


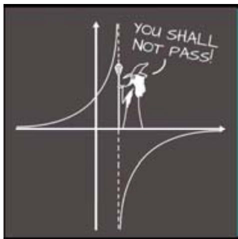


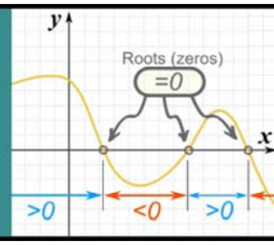
Ex.) Determine the equations for the following:

a) Roots at x = 3, x = 4, x = -7 and it passes through (2, 54).

b) Roots at x = 4 multiplicity of 3, x = 1 multiplicity of 2, and passes through (0, 32).

Oct 19-1:38 PM

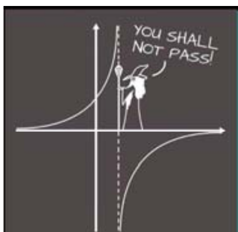




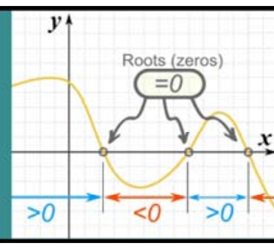
c) Roots at x = 0, -4, 2(mult. of 2) and passes through (-2,128).

Worksheet

Oct 19-1:41 PM

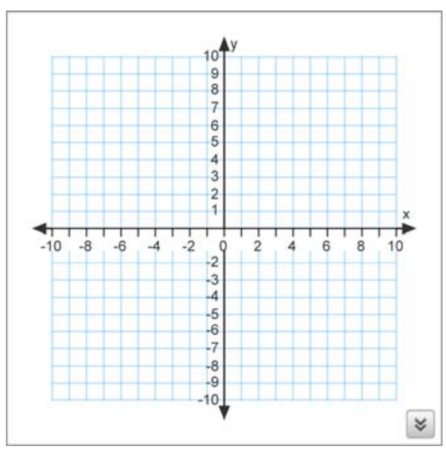


Unit 3: Polynomial, Radical, and Rational Functions



3.6 Transformations of Radical Functions

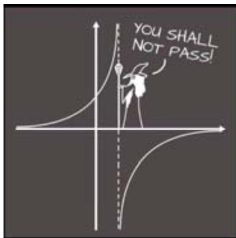
Graph $f(x) = \sqrt{x}$.

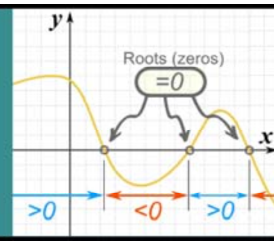


Recall: Transformations

<u>Function Notation</u>   <u>Equation</u>

Oct 19-1:43 PM



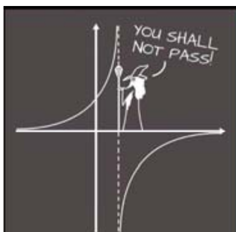


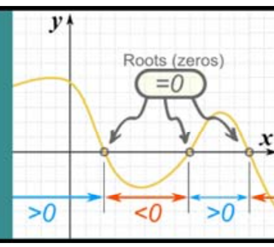
Ex.) For each of the following, describe the transformations and state domain, range, x-int, and y-int.

a)

b)

Oct 19-1:47 PM



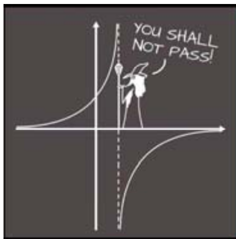


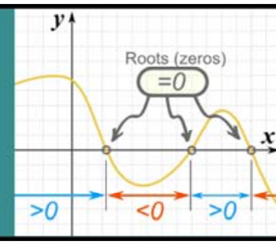
c)

d)

e)

Oct 19-1:48 PM



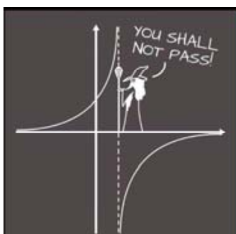


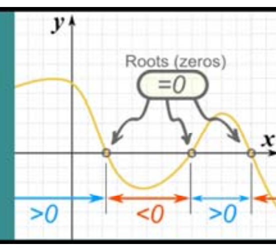
f)

g)

h)

Oct 19-1:48 PM



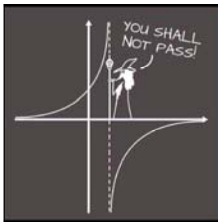


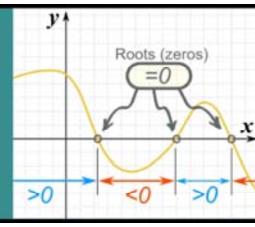
Ex.) Given the graph, determine the radical function.

a)

b)

Oct 19-1:50 PM



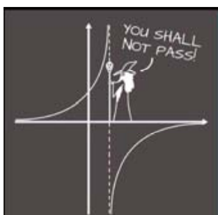


c)

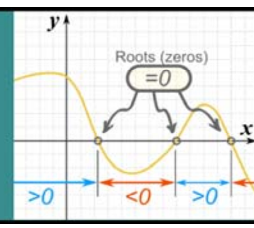
d)

Pg. 72 # 2, 5, 10, 12.

Oct 19-1:51 PM



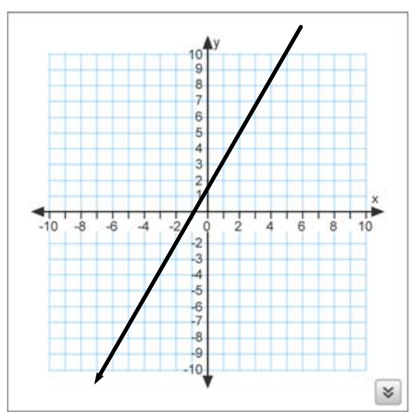
Unit 3: Polynomial, Radical, and Rational Functions



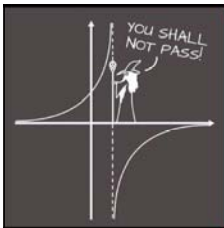
3.7 Graphing the Square Root of a Function

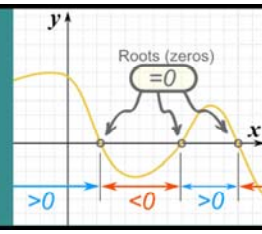
Graph $f(x) = 2x + 1$ and $\sqrt{f(x)}$

Domain:		
Range:		
X-int:		
Y-int:		
Invariant Points:		

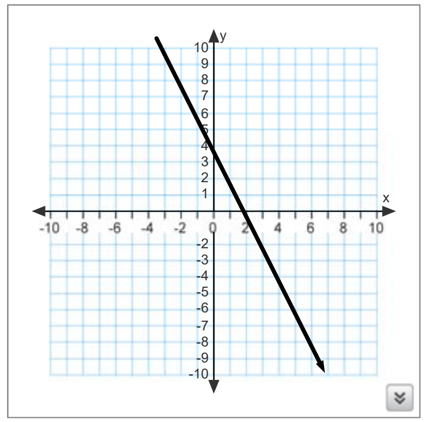


Oct 19-1:53 PM



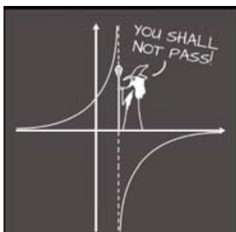


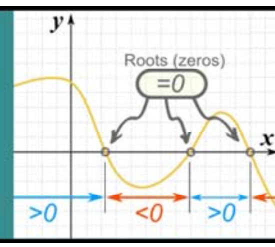
Graph f(x) = 3 - 2x and $\sqrt{f(x)}$



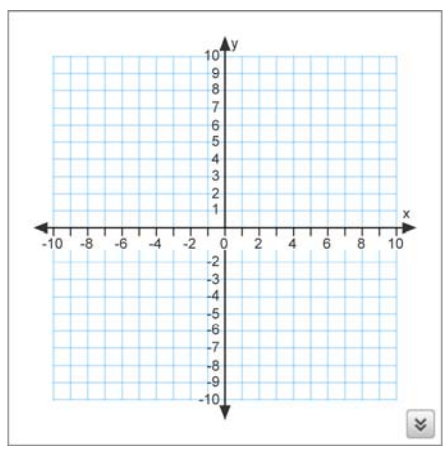
Domain:		
Range:		
X-int:		
Y-int:		
Invariant Points:		

Oct 19-1:59 PM



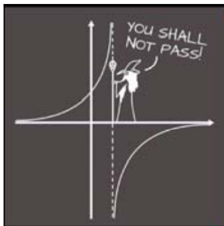


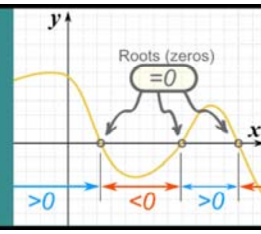
Graph f(x) = x^2 - 4 and $\sqrt{f(x)}$



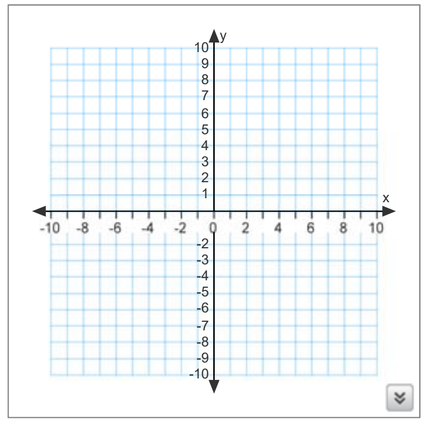
Domain:		
Range:		
X-int:		
Y-int:		
Invariant Points:		

Oct 19-2:00 PM



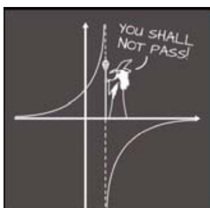


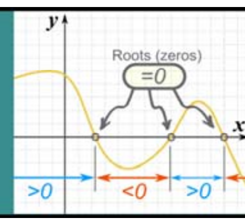
Graph $f(x) = x^2 + 5$ and $\sqrt{f(x)}$



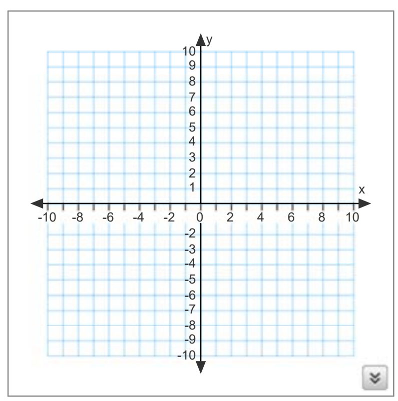
Domain:		
Range:		
X-int:		
Y-int:		
Invariant Points:		

Oct 19-2:01 PM





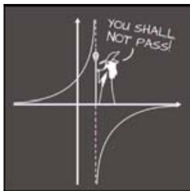
Graph $f(x) = x^2 - x - 6$ and $\sqrt{f(x)}$



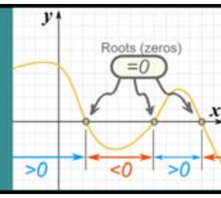
Domain:		
Range:		
X-int:		
Y-int:		
Invariant Points:		

Pg. 86 # 3, 5, 6, 8, 10.

Oct 19-2:02 PM

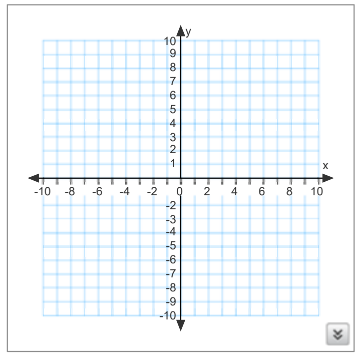


Unit 3: Polynomial, Radical, and Rational Functions



3.8 Rational Functions

Graph f(x) = x and the reciprocal:

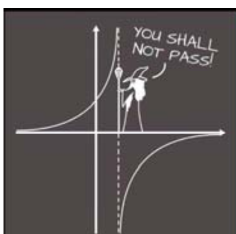


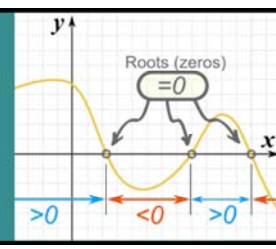
Domain:		
Range:		
X-int:		
Y-int:		
Invariant Points:		
Asymptotes:		

Vertical Asymptote - make the denominator equal to zero

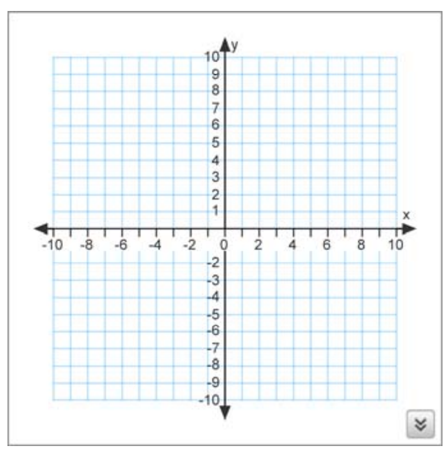
Horizontal Asymptote - divide the coefficients of the highest degree terms

Oct 19-2:04 PM



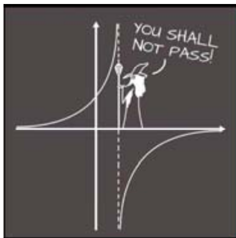


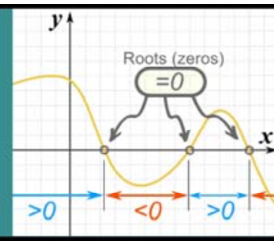
Graph $f(x) = \dfrac{4x - 5}{x - 2}$



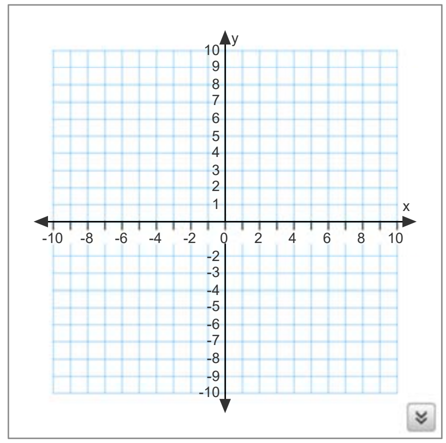
Domain:	
Range:	
X-int:	
Y-int:	
Asymptotes:	

Oct 19-2:33 PM



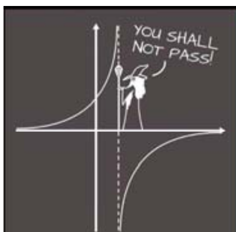


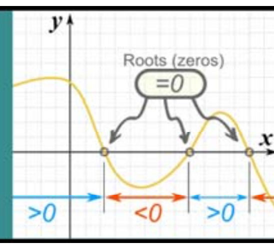
Graph $f(x) = \dfrac{2x + 2}{x - 4}$



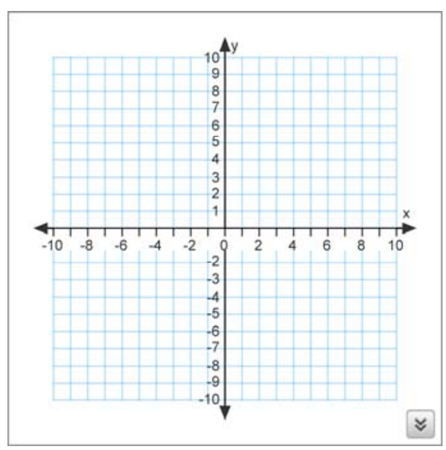
Domain:	
Range:	
X-int:	
Y-int:	
Asymptotes:	

Oct 19-2:34 PM



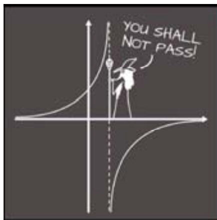


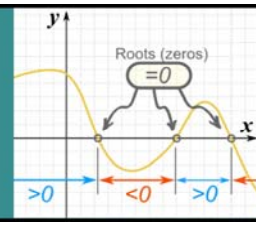
Graph $f(x) = \dfrac{3}{x^2 - 10x + 25}$



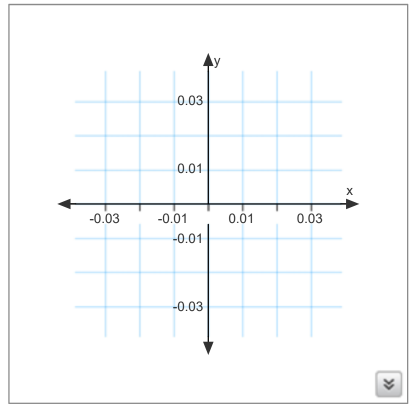
Domain:	
Range:	
X-int:	
Y-int:	
Asymptotes:	

Oct 19-2:34 PM





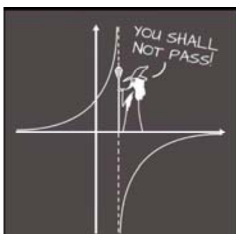
Graph $f(x) = \dfrac{2x^2 - 5x - 3}{x^2 - 1}$

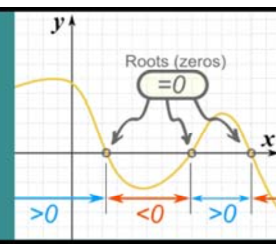


Domain:	
Range:	
X-int:	
Y-int:	
Asymptotes:	

* It is possible to cross a horizontal asymptote on occasion. Vertical asymptotes are not crossed.

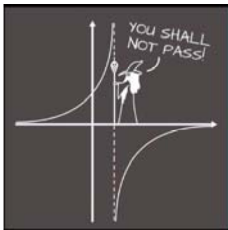
Nov 15-9:08 AM

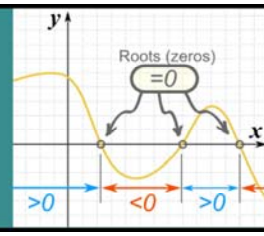




Other possible graphs(but not all possibilities):

Nov 6-1:25 PM



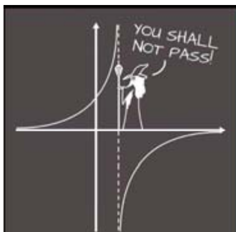


Ex.) a. List the transformations required to transform the mother function $y = \frac{1}{x}$ to $y = \frac{5}{x + 2} - 6$.

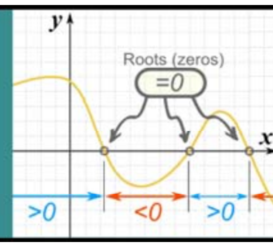
b. Using a common denominator, rewrite the function as one fraction.

Pg. 442 # 1, 3, 4, 5, 8.

Nov 6-1:26 PM



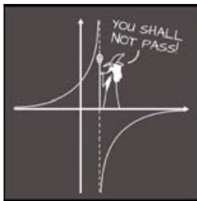
Unit 3: Polynomial, Radical, and Rational Functions

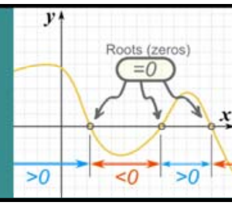


3.9 Analyzing Rational Functions

Point of Discontinuity:

Oct 19-2:37 PM



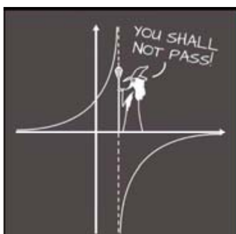


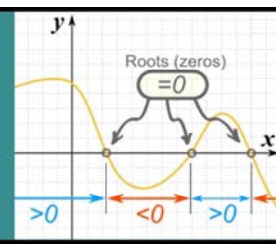
Ex.) Determine the asymptotes, the x, y-intercepts, and the point of discontinuity for the following:

a) $y = \dfrac{x^2 - 5x + 6}{x - 3}$

b) $y = \dfrac{x^2 - 2x}{4 - 2x}$

Oct 19-2:42 PM

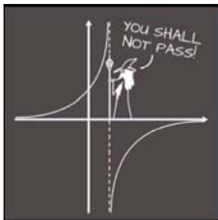


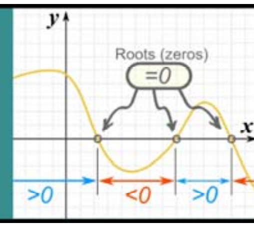


c) $y = \dfrac{x^2 + 7x + 6}{x^2 + 8x + 12}$

d) $y = \dfrac{2x^2 - 15x + 7}{x - 7}$

Oct 19-2:52 PM





Ex.) **Determine** the equation in factored form of the rational function with HA y = 2, VA x = 0, and point of discontinuity (1 , 5).

Pg. 451 # 4, 8ab, 9.