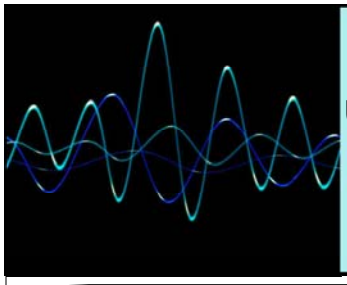

Unit 4: Oscillatory Motion and Mechanical Waves

4.1 Simple Harmonic Motion

Oscillatory Motion - repetitive back and forth motion (ie. wings, strings vibrating, electrical current)

Cycle - one complete oscillation

Period - the amount of time it takes to complete one revolution

Frequency - oscillations/cycles per second

$$f = 1/T$$

f = frequency in hertz (Hz) or cycles/second

T = period (seconds)

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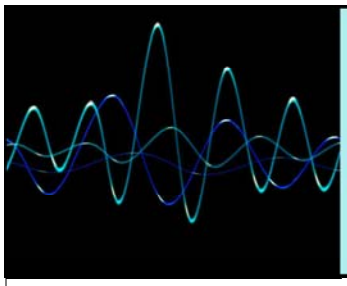
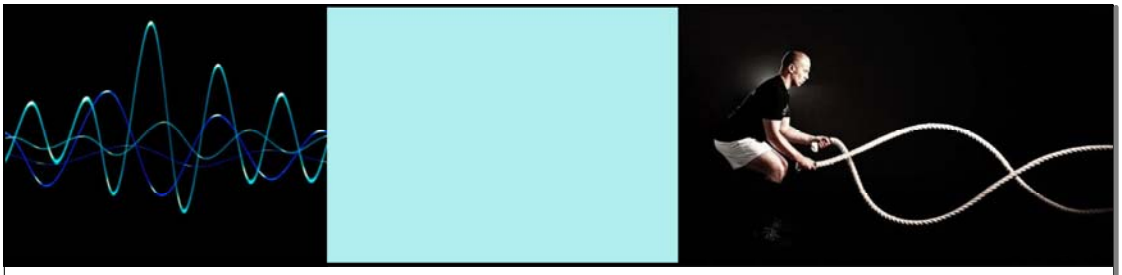


Ex.) The wings of a Canadian Goose flap 200 times per minute.

a) What is the frequency of the flap?

b) What is the period of the flap?

Jan 27-1:48 PM

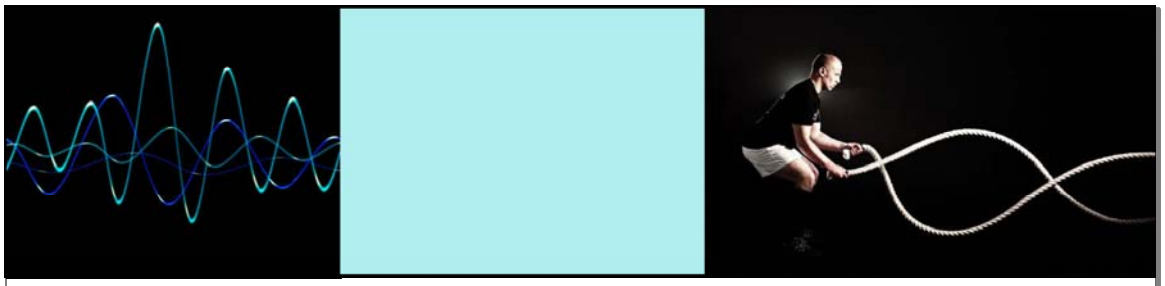


An oscillating object needs a force to keep it going. When the force comes from inside the system it is called a restoring force. Any oscillating system which has a restoring force acting against the displacement to keep an object in motion is a simple harmonic oscillator and exhibits simple harmonic motion.

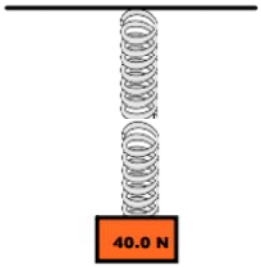
One example is this mass sliding along a frictionless surface attached to a spring. The spring is providing the restoring force to this system:

Analyze this motion in further detail on page 355. Then look at a vertical set up on page 356.

Jan 27-1:50 PM



Ex.) Two springs are hooked together and one end is attached to a ceiling. Spring A has a k = 25 N/m and spring B has a k = 60 N/m. A mass weighing 40.0 N is attached to the free end of the spring system. What is the total displacement of the mass?



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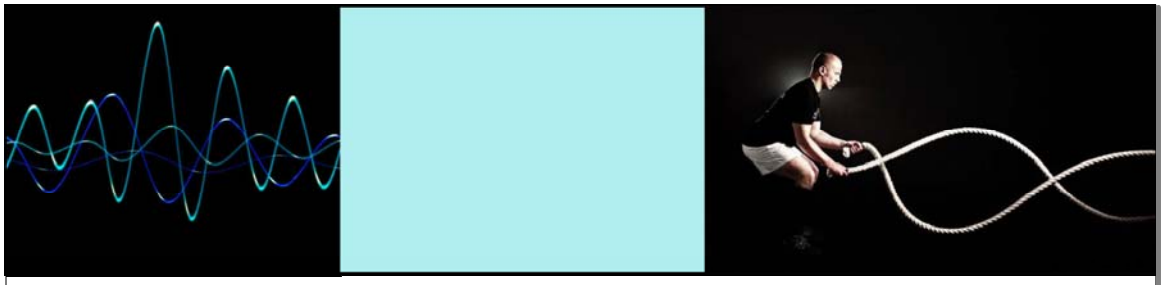

The following rules apply for objects undergoing Simple Harmonic Motion:

- there is a restoring force acting in the opposite direction of the displacement (a force acting opposite of the movement to pull the object back and keep it oscillating)

- at the maximum displacements, the restoring force is at its maximum. This displacement is called the oscillators amplitude. The velocity at this point is zero.

- at equilibrium, the restoring force is zero and the velocity is at its maximum.

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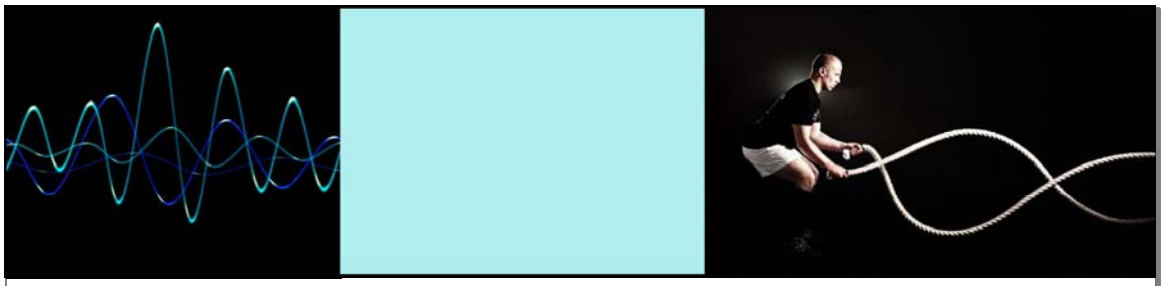
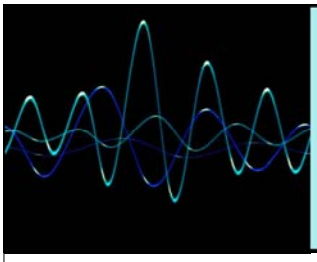


Ideal Pendulum - swings through a small angle, has no friction, and has all mass concentrated at the bob

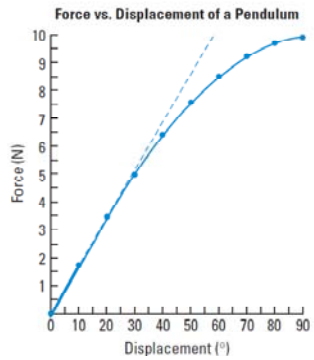
The restoring force in a pendulum is a component of the force of gravity acting opposite the displacement of the bob. Analyze on page 360.

$F_r = F_g \sin\theta$

Jan 27-2:02 PM

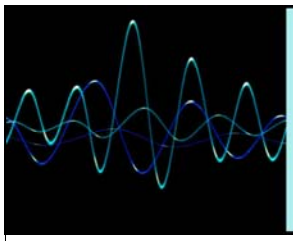


Note: For the pendulum to be a true simple harmonic oscillator, its graph of restoring force versus displacement should be linear, as the dotted line suggests. After 15o , its line departs from the straight line, and its motion can no longer be considered SHM.



Questions: Pg. 365 # 1-6.

Jan 27-2:06 PM



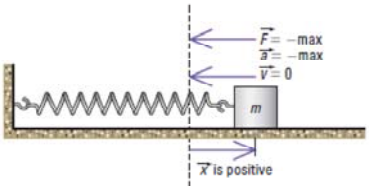
Unit 4: Oscillatory Motion and Mechanical Waves

4.2 Position, Velocity, Acceleration and Time of SHM

Today we want to use the two systems we studied last day (a mass-spring system and a pendulum) to mathematically approximate the position, velocity, acceleration and period of these movements.

1. Mass-Spring System

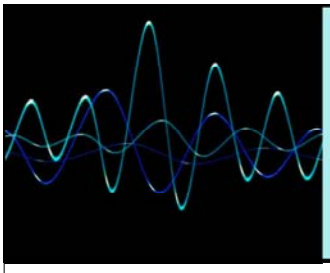
Finding Acceleration



We know that the max. acceleration occurs when the mass is at its amplitude (max. displacement).

Note: these same principles would apply to a vertical mass-spring system

Jan 27-2:10 PM



When the mass is at its amplitude, we can make the restoring force in the spring equal to Newton's Second Law:

$$\vec{F}_s = \vec{F}$$

$$-k\vec{x} = m\vec{a}$$

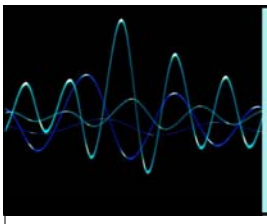
$$\boxed{\vec{a} = \frac{-k\vec{x}}{m}}$$

where:

$\vec{a}$ = maximum acceleration of the block (m/s²)
$\vec{x}$ = maximum displacement of the block (m)
k = spring constant (N/m)
m = mass of block (kg)

Note: only applies to max. acceleration because it is uniform. Acceleration at other points is not uniform and is beyond the scope of Physics 20.

Jan 27-2:15 PM



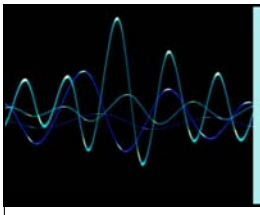
Ex.) In a mass-spring system, a 1.55 kg mass oscillates horizontally when attached to a spring of k = 15 N/m. If the amplitude of the oscillations is 0.75 m, what is the:

a) Magnitude of the max. acceleration of the mass?

b) Direction of acceleration?

c) Max. restoring force acting on the mass?

Jan 27-2:18 PM



Finding Velocity

- max. velocity occurs when the mass is at equilibrium and the force is zero.

When the mass is pulled back, all the energy is PE:

$E_{Pspring} = 1/2kx^2$

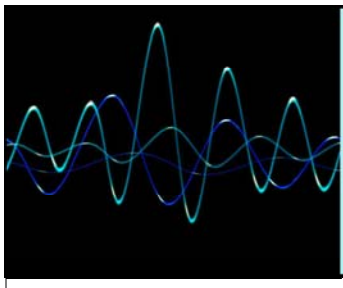
When the mass is at equilibrium, all the PE has turned into KE:

$E_K = 1/2mv^2$

Solve: $E_{pspring} = E_k$

$\vec{v}$ = maximum velocity of mass (m/s)
$\vec{x}$ = maximum displacement (amplitude) (m)
k = spring constant (N/m)
m = mass (kg)

Jan 27-2:19 PM



Ex.) Continued...In the previous system, what will; the max. speed of the mass be?

m = 1.55 kg

k = 15 N/m

x = 0.75 m

Jan 27-2:24 PM

Finding Period

Objects undergoing Uniform Circular Motion can replicate objects in Simple Harmonic Motion.

So recall from Unit 3 that:

$$\vec{v} = \frac{2\pi r}{T}$$

and that:

$$\vec{v} = \vec{x}\sqrt{\frac{k}{m}}$$

Therefore:

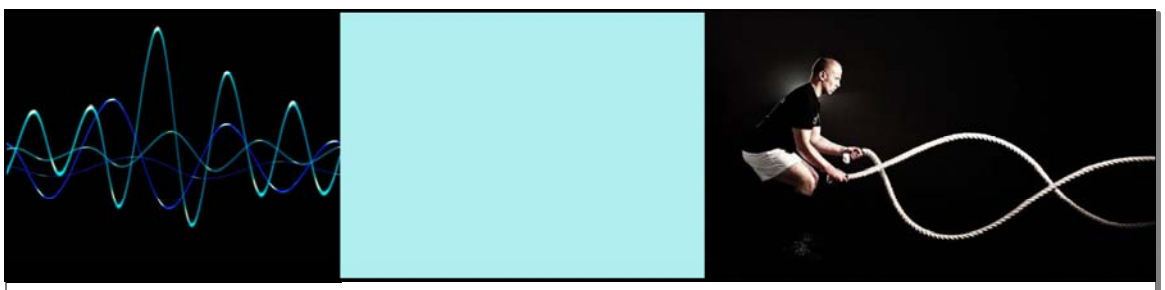
$$v = v$$

$$\frac{2\pi r}{T} = x\sqrt{\frac{k}{m}}$$

If we let x = r

$$\frac{2\pi \cancel{x}}{T} = \cancel{x}\sqrt{\frac{k}{m}} \longrightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

Jan 27-2:25 PM



Formula for Period of a Mass-Spring System

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Where:

T = period (s)

m = mass of oscillator (kg)

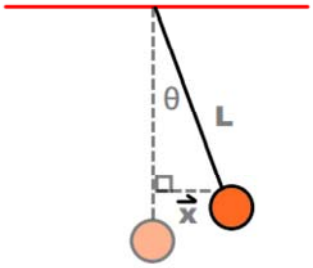
k = spring constant (N/m)

Note: the period of a mass-spring system does not depend on displacement (how far it is pulled back)

Jan 27-2:30 PM

2. An Ideal Pendulum

Since there is no spring constant to apply to a pendulum we cannot use the previous formula. However, we do pull a pendulum back through an angle, θ, so we will use this idea to create an appropriate equation:



where:
L = length of pendulum
$\vec{x}$ = displacement

We could write that:

$$\sin\theta = \frac{\vec{x}}{L}$$

Jan 27-2:32 PM

Formula for Period of a Pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Where:

T - period (s)

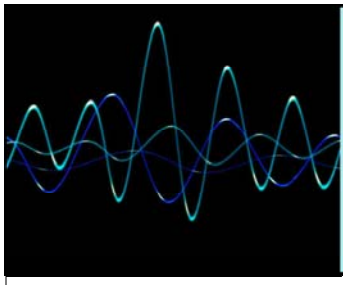
L = length of pendulum from end of string to centre of mass (m)

g - acceleration due to gravity (m/s^2)

Note: period of a pendulum does not depend on mass or amplitude. Only length and gravitational field strength.

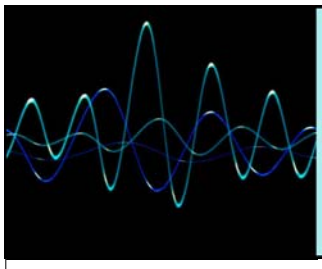
You could perform a simple experiment with pendulums (if you know length and period) to calculate gravitational field strengths on any planet without any other equipment.

Jan 27-2:37 PM



Ex.) An astronaut lands on a planet and constructs a simple pendulum with length 5.5 m to determine the acceleration due to gravity. She measures the period of the pendulum to be 6.7 s. What is the gravitational field strength of the planet?

Jan 27-2:41 PM



Simple Harmonic Motion in Real Life: <u>Resonance</u>

resonance - vibrating or oscillating system

forced frequency - a force is added to an oscillator to keep it resonating

Eg. Swinging on a swing, analog clock (electrically charged quartz crystal provides force to keep gears in time)

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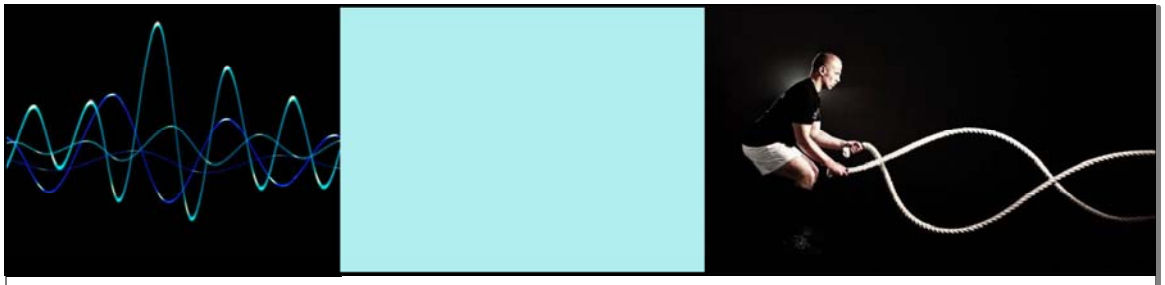


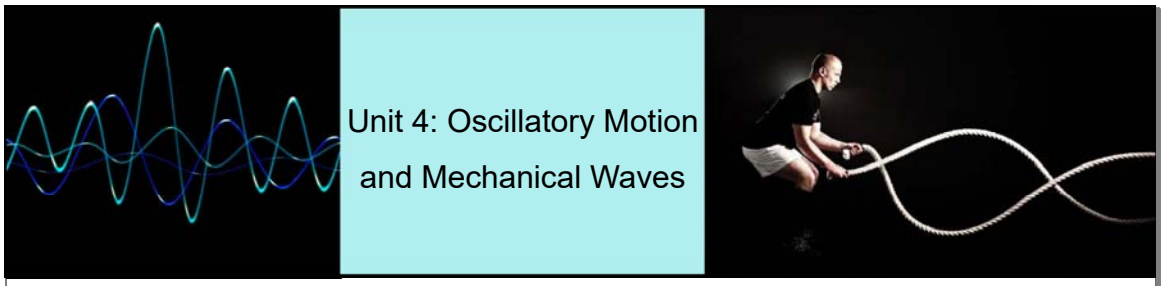
Resonance Disasters:

- 1940 Tacoma Narrow's Bridge destroyed by a gale force wind.
- 1850 Angers, France bridge being marched upon by soldiers in unison.
- Mexican mid rise building during 1985 Earthquake.

https://www.youtube.com/watch?v=j-zczJXSxnw

Questions: Pg. 390-391 # 11-26.

Jan 27-3:05 PM



4.3 Intro to Wave Theory

Wave - the regular transportation of energy without the permanent displacement of matter

- waves are regular because the particles which make them up oscillate in Simple Harmonic Motion, returning to their equilibrium positions
- waves transport energy (ie. heat, energy in light, sound energy, energy in water waves, etc.)
- waves don't displace matter (as the particles eventually return to equilibrium

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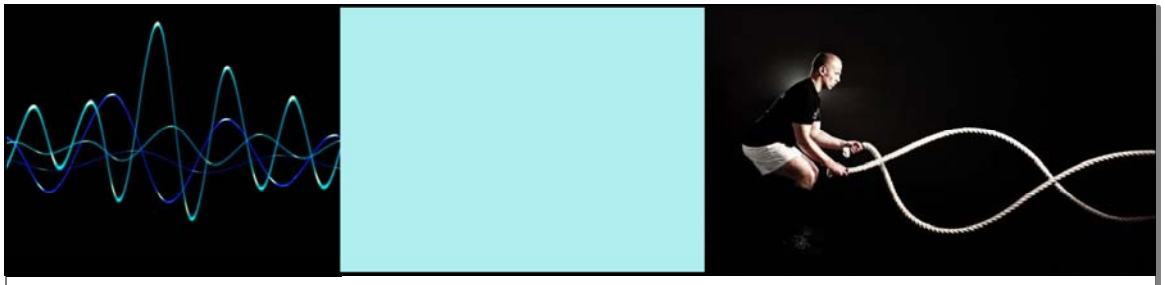

Two Main Types of Waves:

1. **Mechanical Waves** (Physics 20)

   - water waves, sound waves, waves on springs/strings

   - Mechanical waves **need a medium** to travel through like water, air, string, etc.

2. **Electromagnetic Waves** (Physics 30)

   - light, radio/tv/satellite rays, microwaves, UV waves

   - **do not need a medium** to travel through (can go through the vacuum of space)

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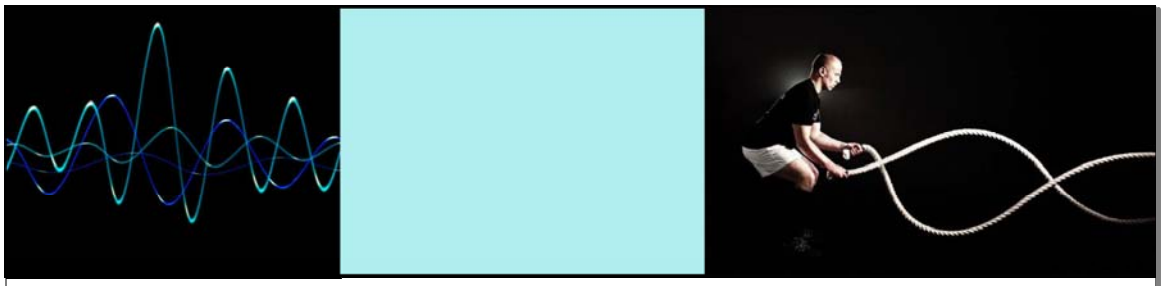
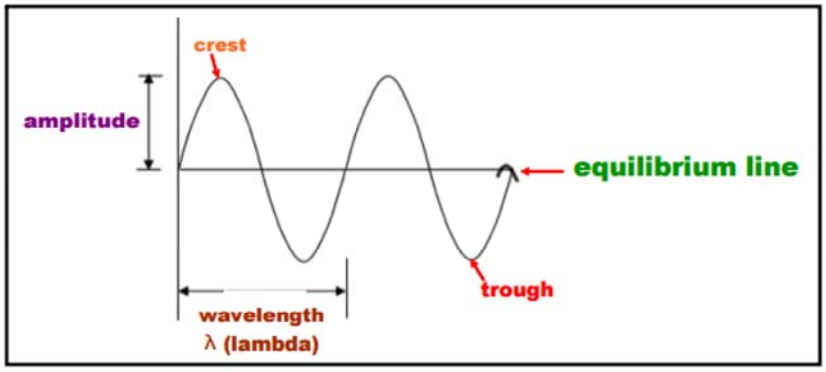


A Sinusoidal Curve is used to draw waves:



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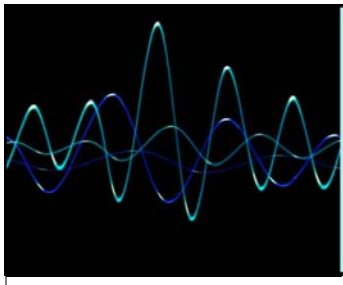

Terminology:

- equilibrium - the initial position of the medium
- crest - the portion of the waves above equilibrium
- trough - the portion of the waves below equilibrium
- amplitude - the distance between the crest and equilibrium or between the trough and equilibrium (m)
- wavelength - the distance between two repeating parts of the wave, symbol λ - "lambda" (m)
- pulses - individual crests and troughs
- wave train - collection of pulses

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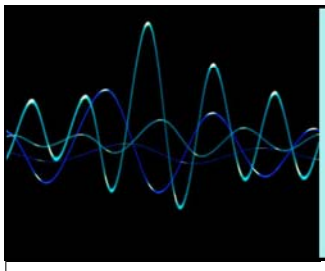

We study two types of wave motion in Physics 20:

1. Transverse Waves: a wave where the particles vibrate in a direction perpendicular to the propagation of the wave.

2. Longitudinal Waves: a wave where particles vibrate in a direction parallel to the propagation of the wave.

Eg.) In transverse waves particles move up and down while the wave moves right to left (ie. waves in string, ripples on water, electromagnetic waves, "the wave").

Eg.) In longitudinal waves particles move right and left while the wave moves right and left (ie. sound waves). Longitudinal waves are made up of areas of high particle density (compressions) and low particle density (rarefaction).

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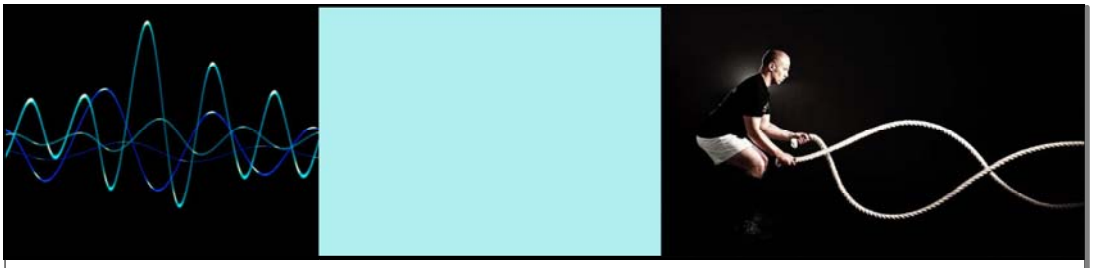


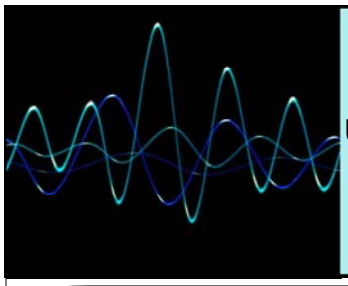
Sometimes we have both transverse and longitudinal waves...Earthquakes are an example of this.

http://www.classzone.com/books/earth_science/terc/content/visualizations/es1002/es1002page01.cfm?chapter_no=visualization

Activity: Pulses in a Spring

Read Pg. 401-410.

Jan 28-8:02 AM



Unit 4: Oscillatory Motion and Mechanical Waves

4.4 The Universal Wave Equation

In Physics 20, waves will move in straight lines at constant velocities. This means we can describe the motion of a line with good ol':

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$

But what is displacement and time of a wave?

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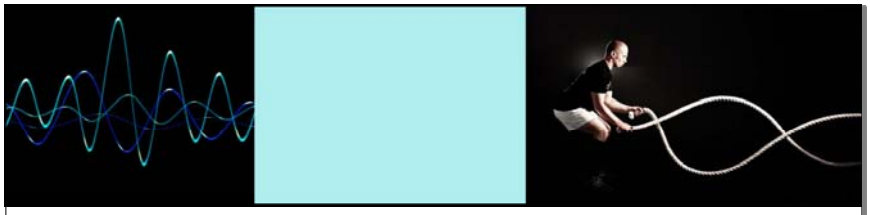
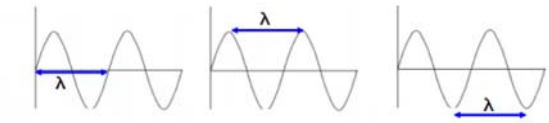




For displacement, we measure the wavelength, λ, of the wave. This is the distance, in m, between any two repeating parts of the wave.

For time, we will use period, T, the amount of time it takes for the wave pattern to repeat.

So...the Universal Wave Equation is:

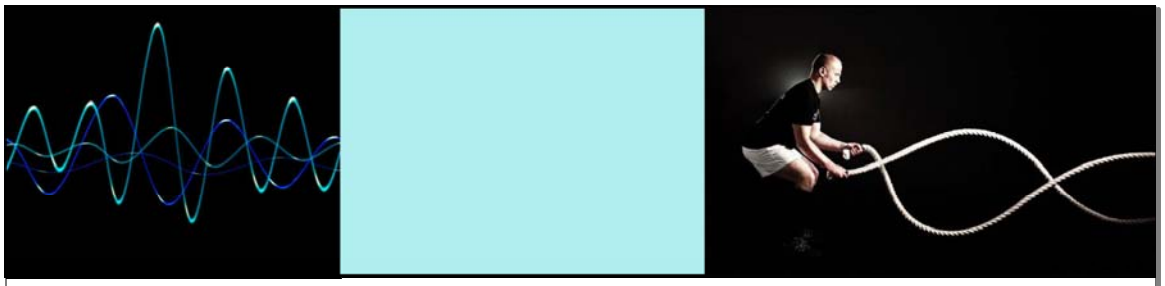
$$v = f\lambda$$

v = velocity (m/s)

f = frequency (Hz or s-1)

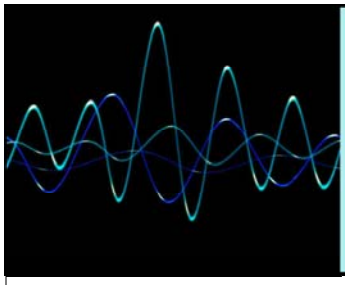
λ = wavelength (m)

Jan 28-8:09 AM



Ex.) The speed of sound in air at 0˚C is 331 m/s. The speed of the same sound increases in air at 20˚C to 343 m/s. If the frequency of the sound is 2.50 x 103 Hz, what is the difference in wavelength?

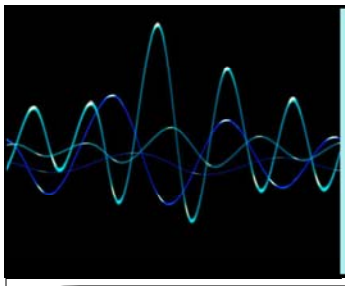
Jan 28-8:25 AM



Ex.) While floating in a tube on a lake, you notice that you bob up and down 4.0 times every 5.0 minutes. You estimate that the distance between the crests is 4.0 m. What is the estimated speed of the water?

Questions: Pg. 410 # 5-9.

Jan 28-8:26 AM



Unit 4: Oscillatory Motion and Mechanical Waves

4.5 Reflection and Interference

When studying waves, they have the ability to:

- reflect (Physics 20)
- refract (Physics 30)
- superimpose (Physics 20)
- diffract (Physics 30)

So far we have seen waves shown as a sinusoidal curve. But ripples in a pond do not look like a sinusoidal curve. That's okay because we can still study these waves in a slightly different manner.

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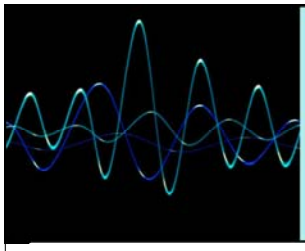

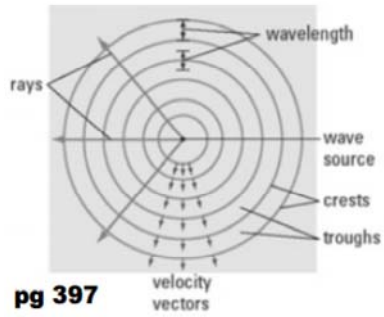

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Notice this wave diagram also has:

- **crests (circular lines)**
- **troughs (halfway between adjacent lines)**
- **wavelength (λ, the distance between crests/troughs)**
- **velocity (direction and speed of wave movement)**

However, this diagram also introduces a new term: rays.

A ray is a line drawn perpendicular to wave fronts or crests. The ray indicates the direction of movement of the wave.

http://www.acoustics.salford.ac.uk/feschools/waves/super.php

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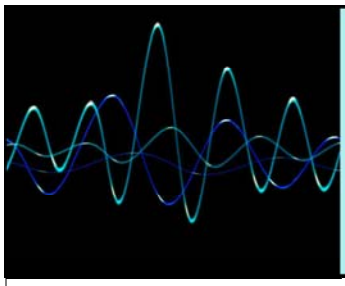
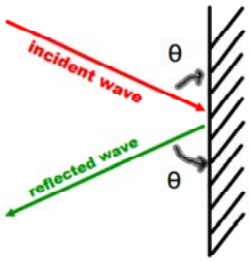


Reflection

When a wave pulse hits a barrier of different density, it will reflect.

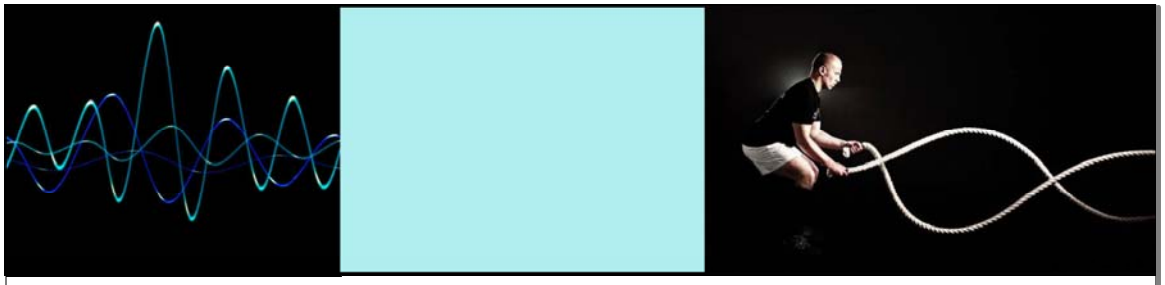
In a relfection, all properties of the wave (λ, v, T) stay the same; only the direction changes.

We call the wave an incident wave before it strikes the boundary and a reflected wave afterwards.

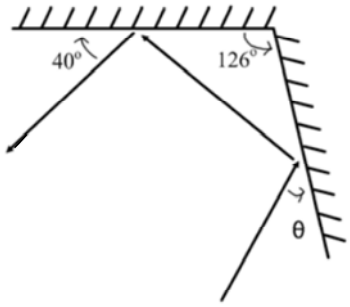


In a wave reflection,
Angle of Incidence = Angle of Reflection

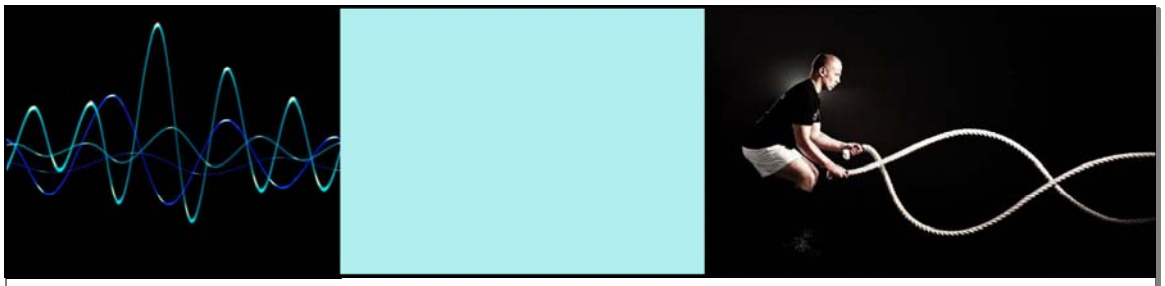
Jan 28-9:00 AM



Ex.) Determine the angle θ below:

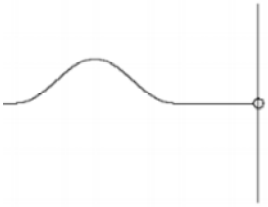


Jan 28-9:05 AM



Waves in a spring or string also reflect:

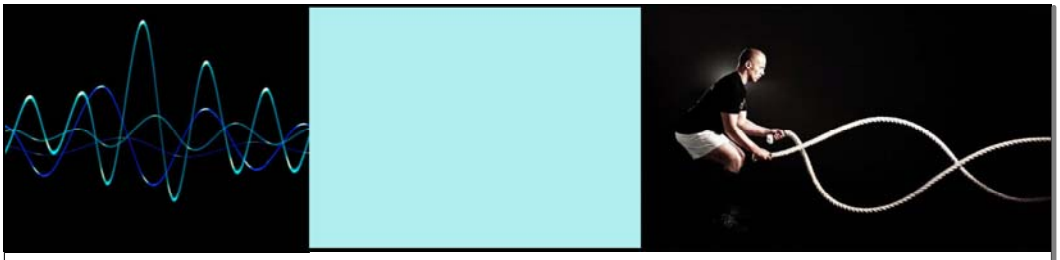
When a crest is incident (hits) a fixed boundary, it reflects and inverts to a trough.



When a crest hits an unfixed boundary, it reflects as a crest.

http://www.acs.psu.edu/drussell/Demos/reflect/reflect.html

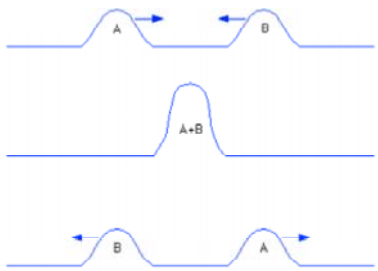
Jan 28-9:06 AM



So what happens if we send two crests at each other? Two troughs? A crest and a trough?

This is called;

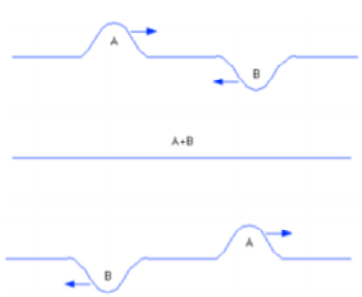
Constructive Interference - overlap of waves to create one wave with a larger amplitude.



Jan 28-9:12 AM

Similarily, when a crest and trough meet, their amplitudes are added. However, this results in a pulse of smaller amplitude (or no amplitude) being created. This is known as destructive interference.

Destructive Interference - overlap of pulses to create a pulse of smaller amplitude.



The idea behind constructive and destructive interference are referred to as **superposition**.

Jan 28-9:14 AM

The effect can be found in wave front waves:

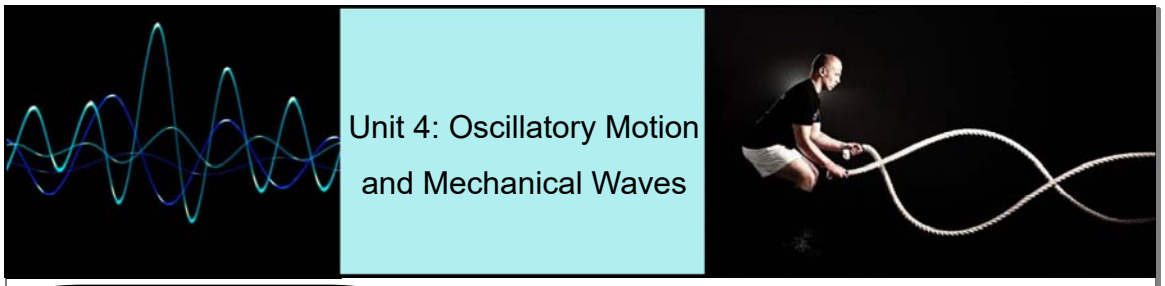
Where the waves overlap, we have constructive interference (the waves are in phase).

Where the waves are not overlapping, we have destructive interference (waves are out of phase).

Activity: Pg. 414-415 (Read the instructions first, do the activity, answer the analysis questions.)

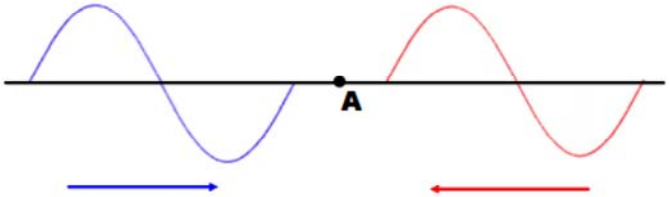
Worksheet: Waves, Resonance, Doppler Effect

Jan 28-9:24 AM



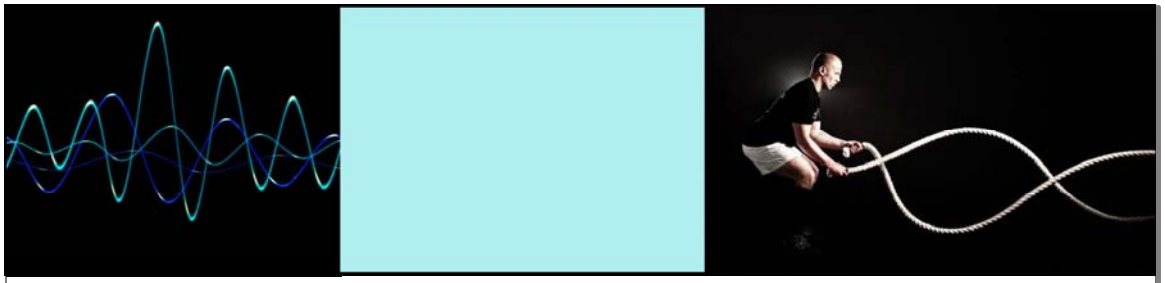
4.6 Acoustic Resonance

Let's look at what happens when we have a group of pulses (a wave train) interfering:

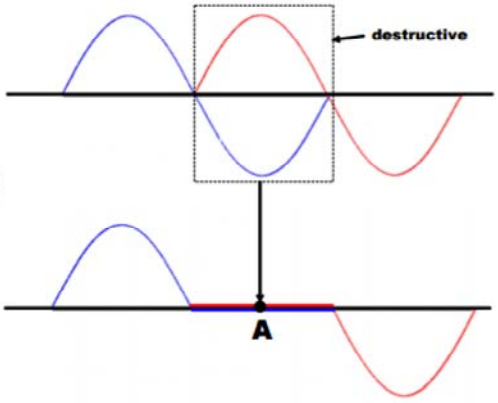


1) The pulses move towards each other and the point A on the spring/string/medium.

Jan 28-9:27 AM

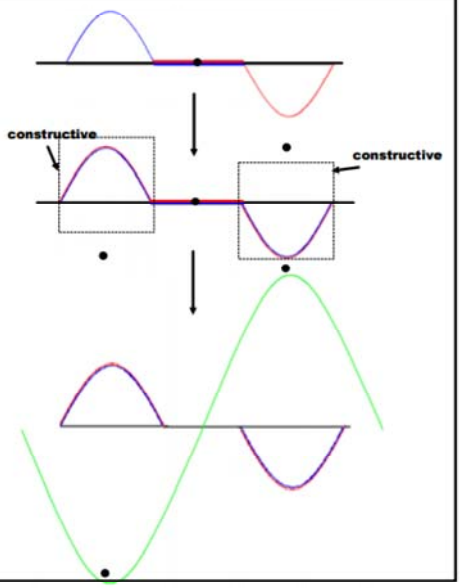


2) The first pulses destructively interfere:



Jan 28-9:29 AM

3) But as the waves pass, the pulses then constructively interfere:



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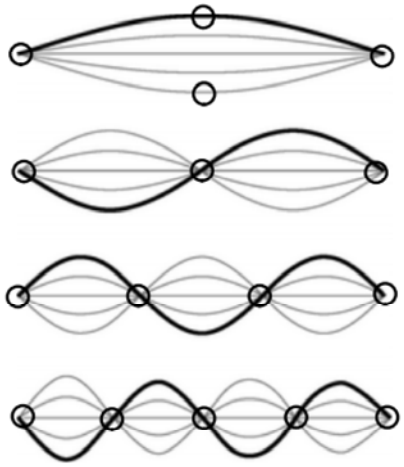
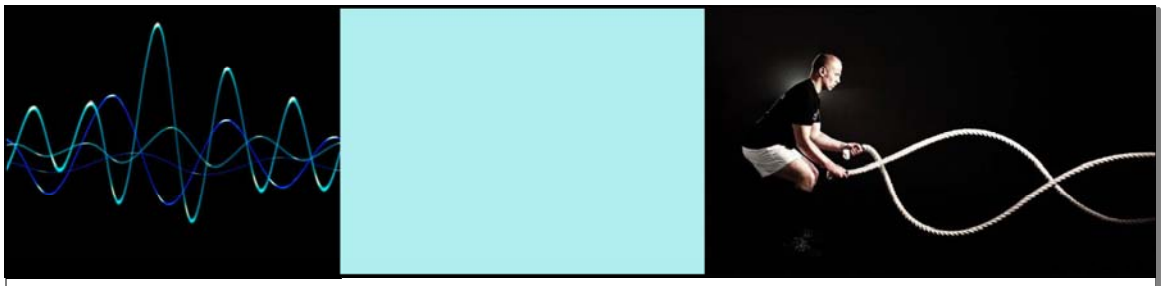
The resulting pattern is called a standing wave. A standing wave appears to oscillate about set points called nodes.



- nodes
- antinodes (the regions of maximum displacement)

Jan 28-9:29 AM



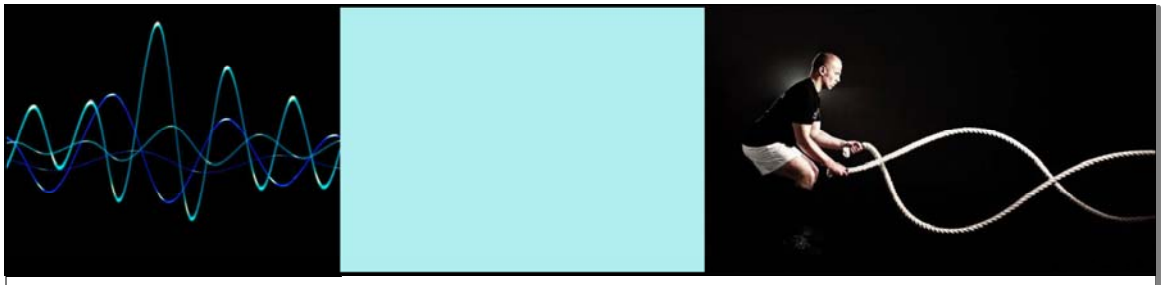
The Sound of Resonance

Resonance - when a vibrating system drives another system to oscillate with greater amplitude

▲ **Figure 8.32** The tone produced when you blow across the top of an open bottle depends on the length of the air column.

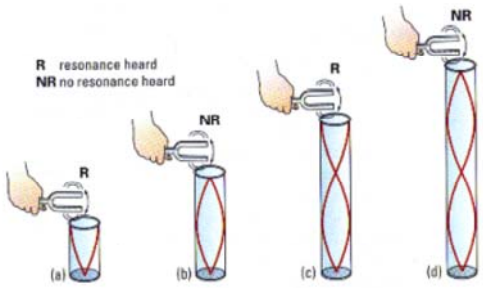
Wind instruments take advantage of resonance to produce music. The air vibrates as a standing wave, producing and antinode in the mouth of the instrument and a node at the bottom of the intrument.

Jan 28-9:32 AM

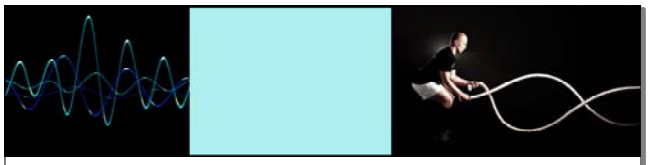


Resonance only occurs in air in an open cylinder if the open end of the cylinder has an antinode and the closed end has a node.

Resonance in air results in a greatly amplified sound wave that you can hear called a harmonic.



Jan 28-9:36 AM



As resonance is easily detected, and occurs when antinodes are at the open end of a cylinder, we can use an open ended air column to determine the speed of sound in air.

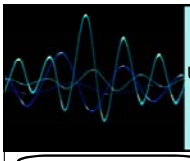
Remember: antinodes occur at 1/4λ and every 1/2λ after that (1/4λ, 3/4λ, 5/4λ, etc.)

Ex.) A tuning fork tuned to middle C (440 Hz) is held at the end of a 1.00 m closed at one end cylinder. The second harmonic is heard in the cylinder. What is the speed of the sound waves around this cylinder?

Questions: Pg. 420 # 1-4.

Add question from the textbook on pg. 420 # 3 or 4 where you use 1/2lambda, 2/2lambda, etc.

Jan 28-9:37 AM



Unit 4: Oscillatory Motion and Mechanical Waves

4.7 The Doppler Effect

https://www.youtube.com/watch?v=Y5KaeCZ_AaY

Recall that frequency is defined as the number of waves passing a point in one second.

So a higher frequency means more waves per second and a lower frequency means fewer waves per second.

Christian Doppler described this effect in 1824.

He realized that when a sound source was moving towards the observer, the waves would be (squashed together", increasing their perceived frequency.

A similar effect will take place when the source moves away; the waves are move "spaced out", causing a lower perceived frequency.

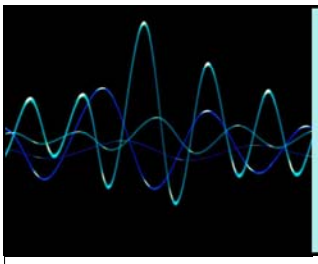
Christian Doppler
(1803-1853)

Random Scientist Facts

Doppler was a man of small stature and was very weak. So much so that he required sick leave after finding teaching a very strenuous job. It also contributed, in part, to him becoming a scientist. His fathers work, stonemasonry, was far to physical for this little man.

http://galileoandeinstein.physics.virginia.edu/more_stuff/flashlets/doppler.htm

Jan 28-9:42 AM



The Doppler Effect Mathematically:

$$f_d = f_s\left(\frac{v_w}{v_w \pm v_s}\right)$$

Where:

fd = perceived freq. (Hz)

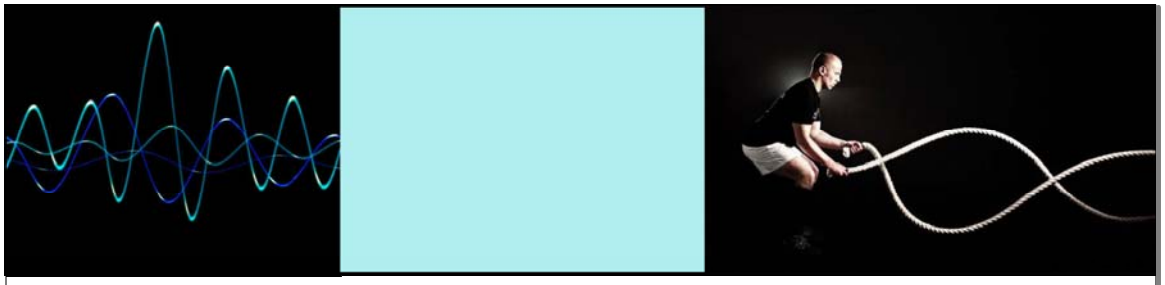
vw = speed of sound in air

vs = speed of source

fs = freq. of source (Hz)

Note: when the source moves towards the observer use subtraction, when the source moves away from the observer use addition.

Jan 28-9:57 AM

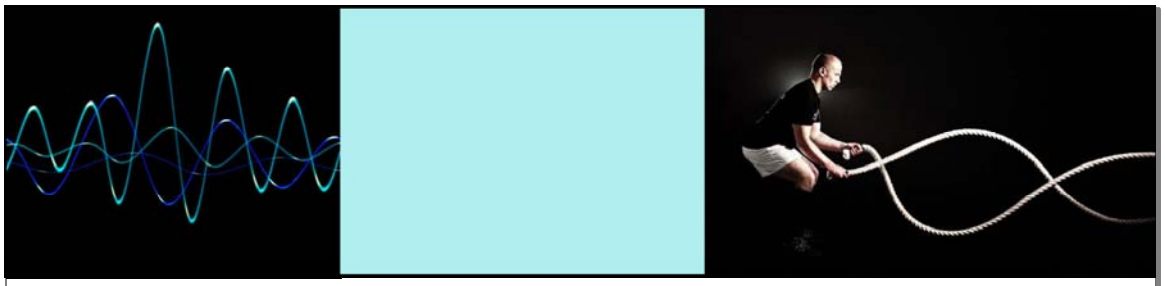


Ex.) A police car has a horn with frequency 220 Hz. If the speed of sound is 340 m/s, and the car is moving at 120 km/h, determine the perceived frequency of the horn:

a) When the car is moving towards you.

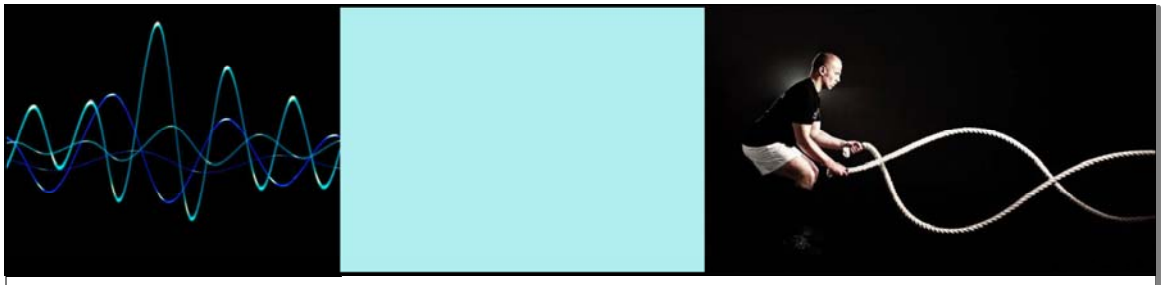
b) When the car is moving away from you.

Jan 28-10:02 AM



Pg. 437 # 18) How fast is a sound source moving toward you if you hear the frequency to be 580 Hz when the true frequency is 540 Hz? The speed of sound in air is 350 m/s. Express your answer in km/h.

Jan 28-10:04 AM

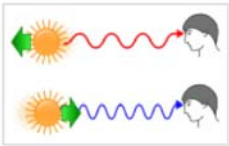


Pg. 437 # 21) If the speed of sound in air is 350 m/s, how fast must a sound source move towards you if the frequency that you hear is twice the true frequency of the sound? What frequency would you hear if the sound source had been moving away from you?

Jan 28-10:05 AM

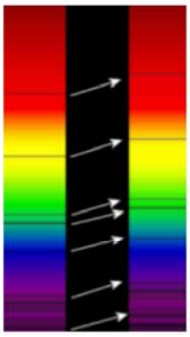
Although Doppler originally intended his equation to be used for sound waves, it applies to other waves, like light waves, as well.

When dealing with light we typically change from frequencies to wavelengths.



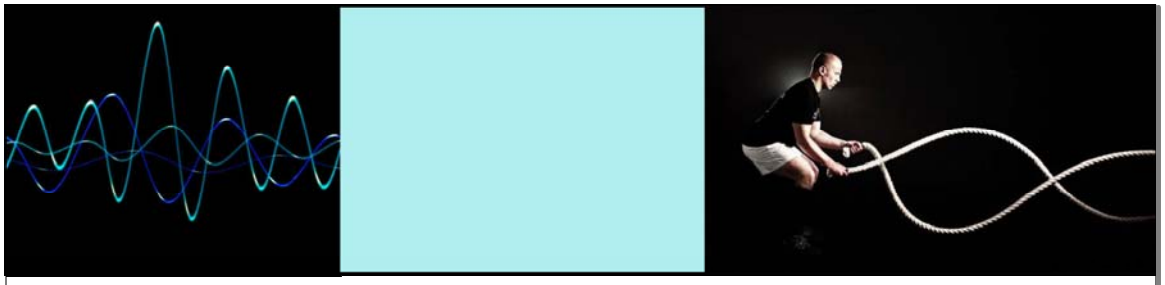
Red shift occurs when the source of light is moving away from the observer.

Blue shift occurs when the source is moving towards the observer.



Astronomers have shown that light from distant stars is red shifted. This has lead to the conclusion that the universe is expanding or moving away from some central point.

Jan 28-10:07 AM



The Sound Barrier

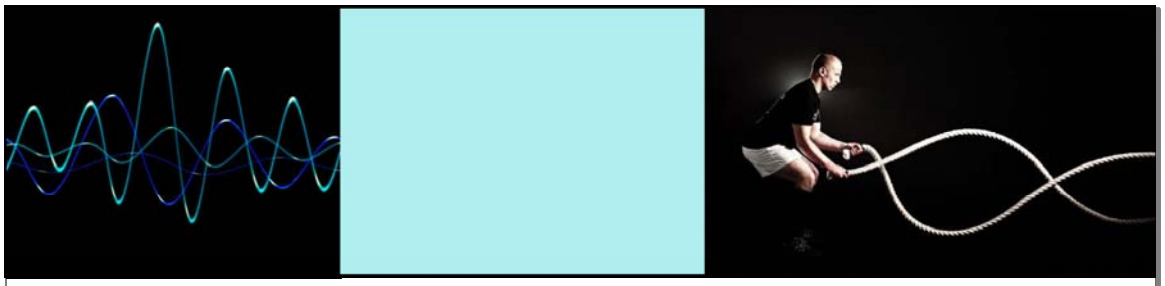
As the speed of sound in air at a particular temperature is fixed, it is possible to exceed this speed.

Objects like a whip, jet plane, or wet dish towel can move faster than the speed of sound, breaking the sound barrier.

If the $v > v_s$, an observer in front of the source will hear a higher frequency than the source is actually producing.

As the speed increases, this frequency also increases.

Jan 28-10:09 AM

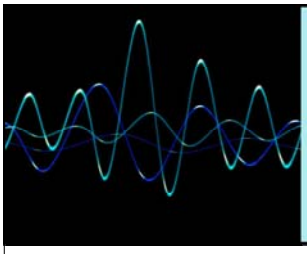


If the $v = v_s$, a build-up of wave fronts occurs. These waves form an area of high pressure which is difficult to overcome (the barrier).

Mach 1

The increase in pressure can cause a cloud to form at the barrier and is perceived not as a frequency but as a loud "thump" as it hits the observer.

Jan 28-10:12 AM



If the $v_s > v$, a sonic boom occurs. The observer will not experience the sound until after the source has passed.

Supersonic

Questions: Finish Unit Assignment.
https://www.youtube.com/watch?v=XmDVvGNtgMg

Jan 28-10:12 AM