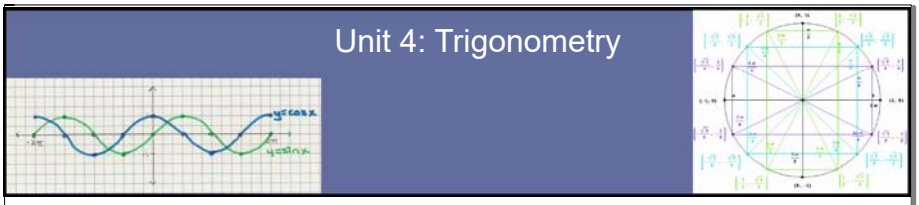
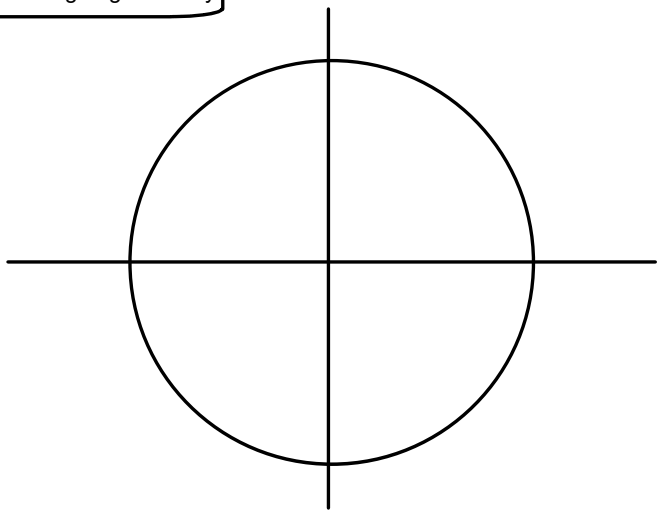
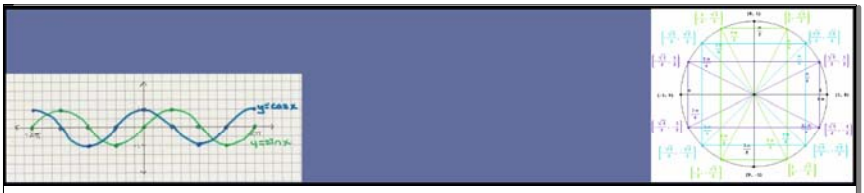


4.1 Discovering Trigonometry



https://betterexplained.com/articles/intuitive-guide-to-angles-degrees-and-radians/

Nov 14-8:04 AM



Converting Between Degrees and Radians:

Conversion Factor:

Ex.) Convert the following:

270°	$\frac{3\pi}{2}$
45°	$\frac{5\pi}{4}$
210°	$\frac{11\pi}{6}$
540°	2.1

Nov 14-8:16 AM

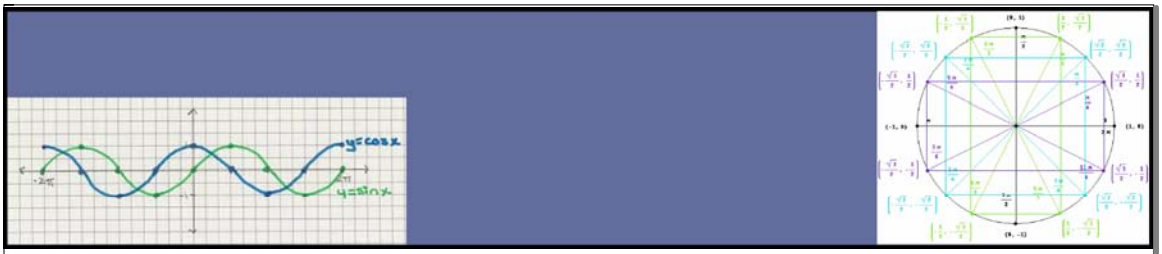
Terminology:

Reference Angles:

Principle Angles:

Co-terminal Angles:

Nov 14-9:01 AM

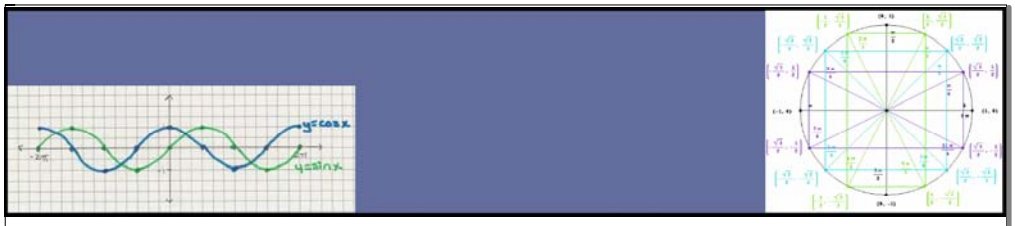


Ex.) Determine one positive and one negative co-terminal angle:

a) $\theta = 120°$

b) $\theta = \frac{5\pi}{4}$

Nov 14-9:03 AM



Ex.) Given the following angles, determine the reference angle:

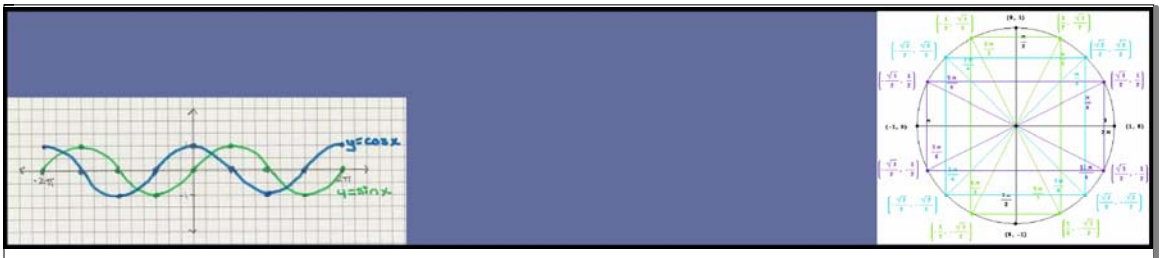
a) 315°

b) 470°

c) $\frac{-5\pi}{6}$

d) $\frac{5\pi}{3}$

Nov 14-9:05 AM

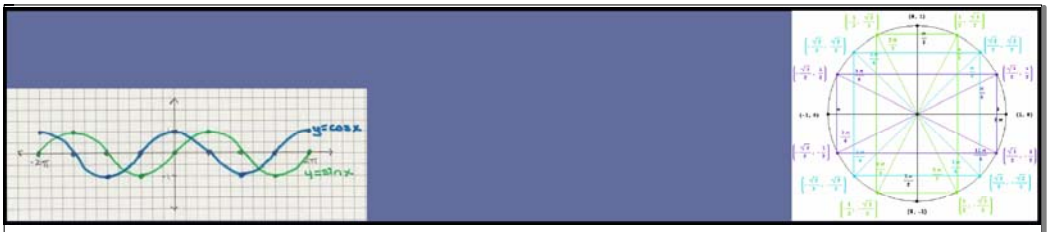


Arc Length:

$$\theta = \frac{a}{r}$$

Ex.) A circle with radius 7 cm, has a central angle of 160° that subtends an arc. What is the length of the arc to the nearest tenth of a centimetre?

Nov 14-9:23 AM



Ex.) An angle of 1.8 subtends an arc 4.5 mm. What is the radius of the circle?

Ex.) A circle with an arc length of 25 m has a radius of 11 m. What is the central angle, to the nearest degree?

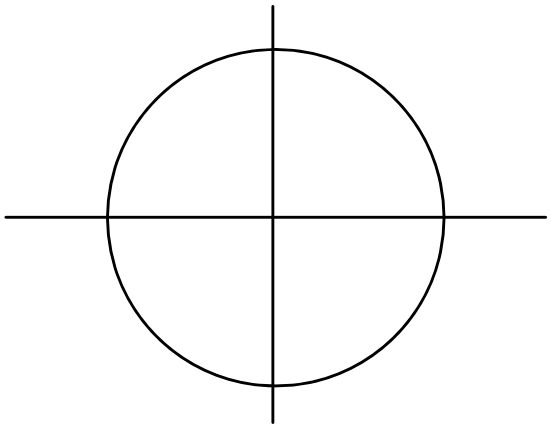
Pg. 175 # 2, 4, 6, 7, 8, 9, 12ab, 13, 14a, 16.

Nov 14-9:23 AM

Unit 4: Trigonometry

https://www.youtube.com/watch?v=HACNCy0clO0

4.2 The Unit Circle



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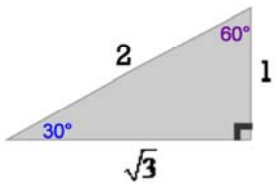
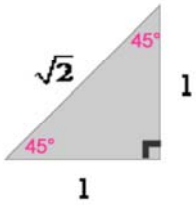
Special Triangles:

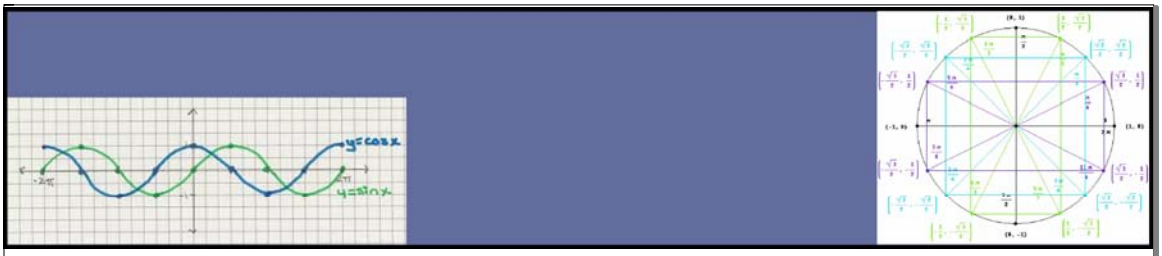
Evaluating Functions of a 30°, 45°, or 60° Angle

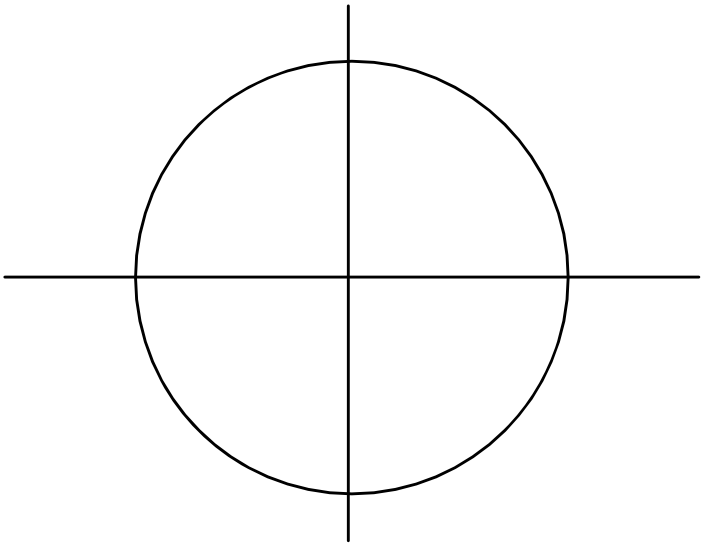




$\sin 30° = \frac{1}{2}$	$\sin 60° = \frac{\sqrt{3}}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$
$\cos 30° = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{1}{2}$	cos ~~sin~~ $45° = \frac{\sqrt{2}}{2}$
$\tan 30° = \frac{\sqrt{3}}{3}$	$\tan 60° = \sqrt{3}$	tan ~~sin~~ $45° = 1$

Nov 14-9:29 AM





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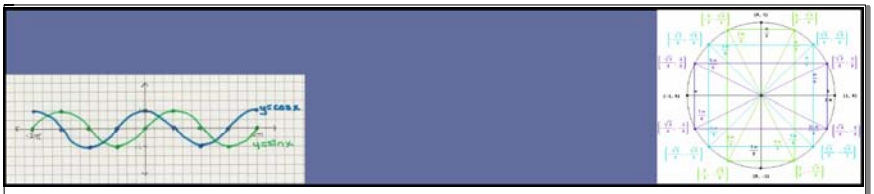


Using the Unit Circle:

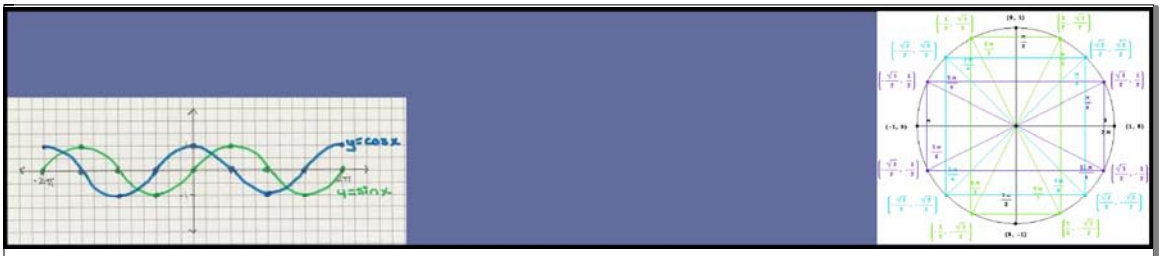
a) tan 45°

b) tan 240°

c) tan π

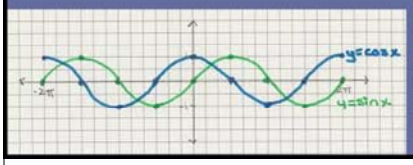
d) tan (π/2)

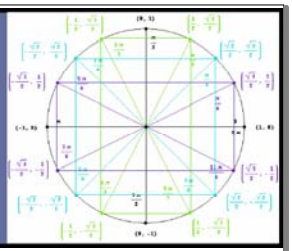
Nov 14-9:31 AM



Ex.) Do the following points exist on the Unit Circle?

Nov 14-9:32 AM

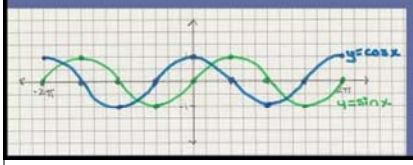




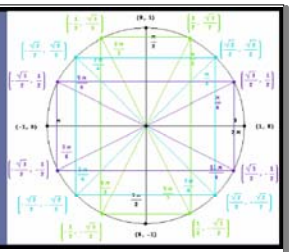
Ex.) The point P(5/6 , y) is on the unit circle. What is/are the value(s) of 'y'?

Pg. 186 # 2, 3, 4, 5, 10.

Nov 14-9:33 AM



Unit 4: Trigonometry

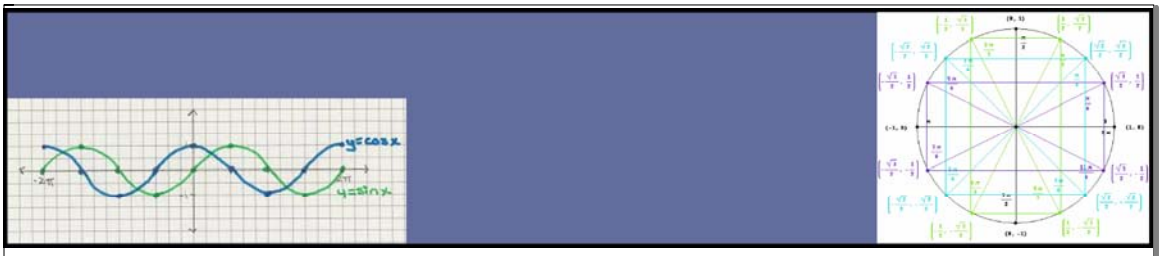


4.3 Trigonometric Ratios

Recall: Good ol' SOH CAH TOA. This gave us the three primary trigonometric ratios. However, they all have companions:

Companions to the primary trig ratios are reciprocal trig ratios:

Nov 14-9:38 AM



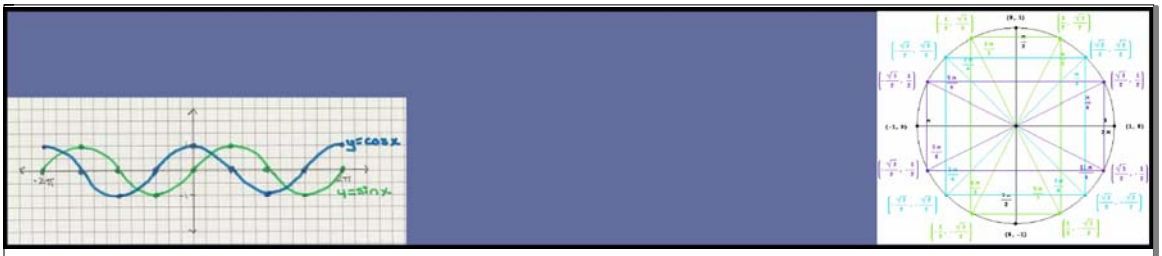
Here's what we get on the formula sheet:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

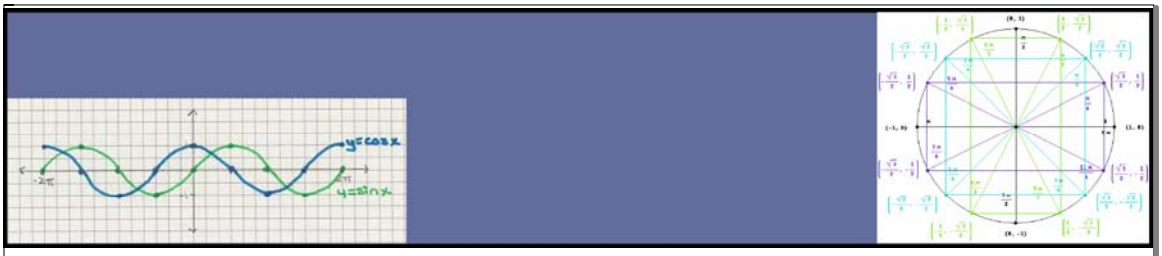
$$\cot\theta = \frac{1}{\tan\theta}$$

Nov 14-9:41 AM



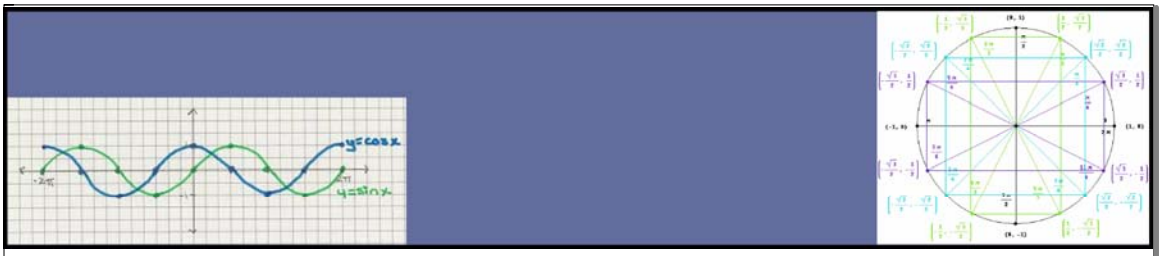
Ex.) Determine the 6 trig ratios for the point P (3 , 4) on the terminal arm of an angle in standard position.

Nov 14-9:42 AM



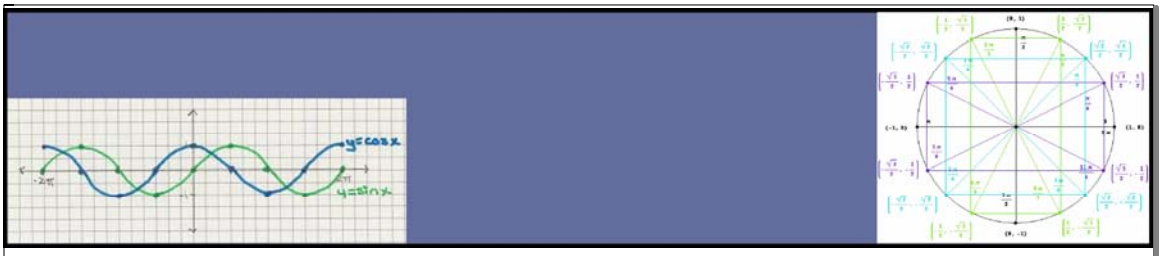
Ex.) Determine the 6 trig ratios for the point P (-3 , 2) on the terminal arm of an angle in standard position.

Nov 14-9:44 AM



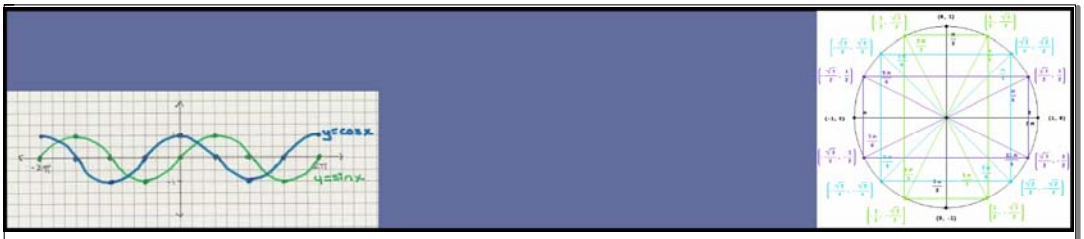
Ex.) If cos θ = 2/5 in quadrant IV, determine the other 5 trigonometric ratios.

Nov 14-9:44 AM



Ex.) If cos θ = 3/5 and tan θ < 0, determine the reference angle and the angle in standard position.

Nov 14-9:45 AM

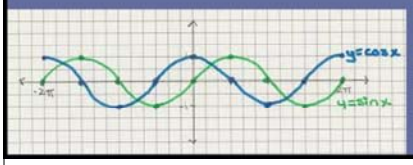


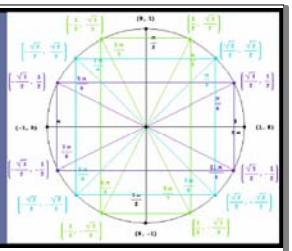
Ex.) Given sec θ = 2/√3, determine θ: -2π < θ < 2π.

Pg. 201 # 1, 3, 8, 9, 10, 12.

Nov 14-9:46 AM

Unit 4: Trigonometry





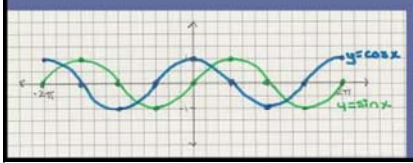
4.4 Solving Linear Trig Equations

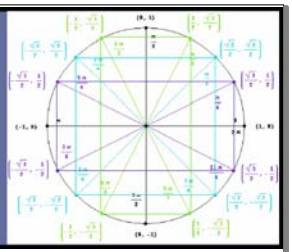
Grade 10: sin θ = 0.3216

Grade 11: CAST

Grade 12: All solutions $0 \le \theta \le 360^{\circ}$ or $0 \le \theta \le 2\pi$

Nov 14-9:49 AM



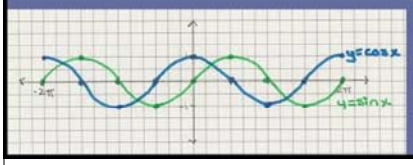


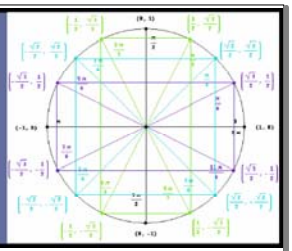
Ex.) Solve the following equations $0 \le \theta \le 360^{\circ}$ and $0 \le \theta \le 2\pi$:

a) $2\sin\theta = -1$

b) $\sin\theta = \sqrt{3} - \sin\theta$

Nov 14-9:52 AM



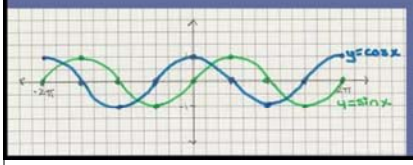


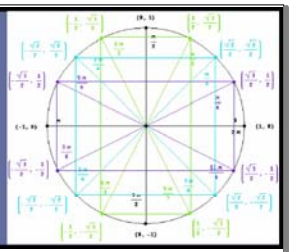
c) $5\cos\theta + 2 = 1 + 3\cos\theta$

d) $3\csc\theta - 6 = 0$

Pg. 211 # 1, 3, 5.

Nov 14-9:54 AM





Unit 4: Trigonometry

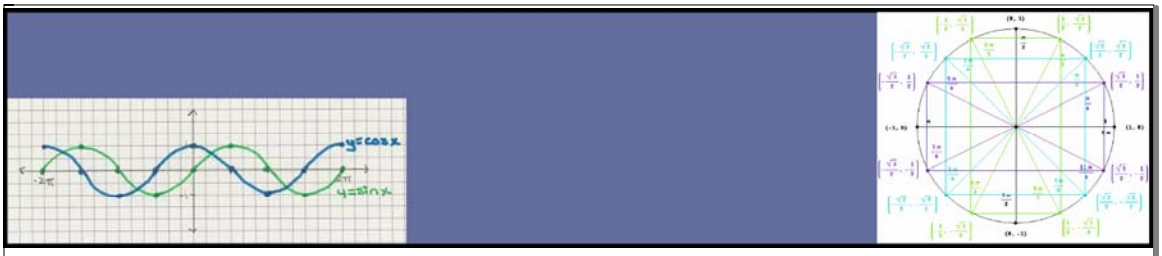
4.5 Solving Quadratic Trig Equations

Factor using the substitution method:

Ex.) Solve the following for $0 \le \theta \le 360^\circ$ and $0 \le \theta \le 2\pi$.

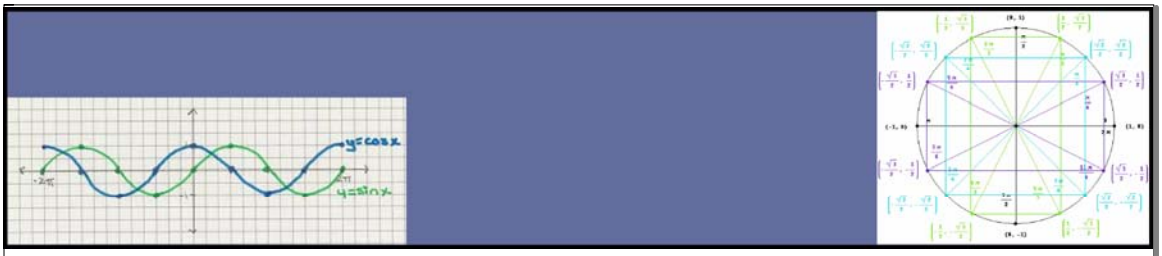
a)

Nov 14-9:58 AM



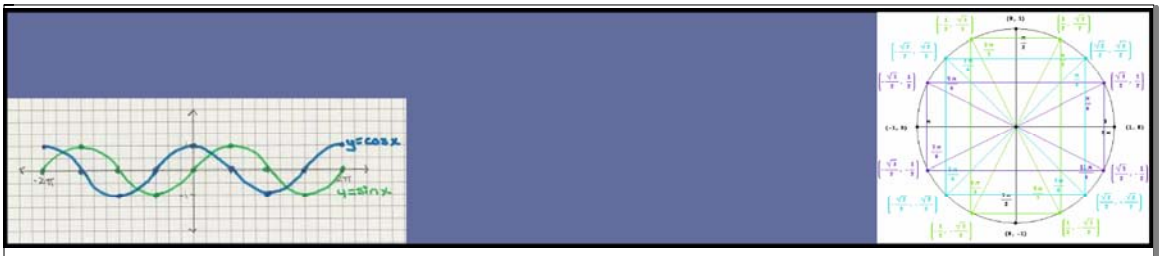
b)

Nov 14-10:00 AM



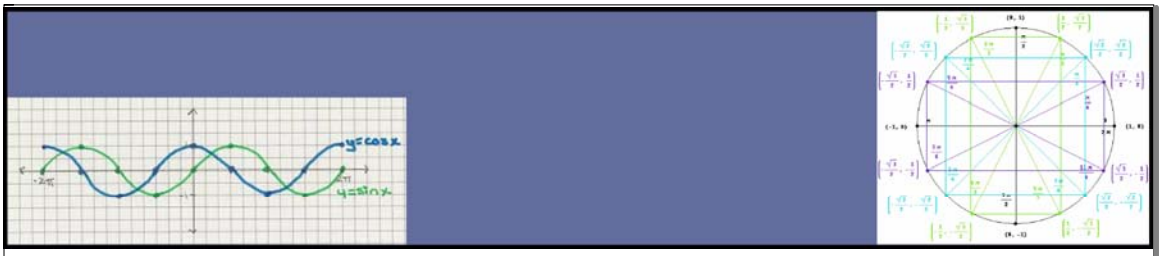
c)

Nov 14-10:00 AM



d)

Nov 14-10:00 AM $0 \leq x \leq 2\pi$



e)

Pg. 212 # 7, 9, 13. ($0 \leq \theta \leq 2\pi$).

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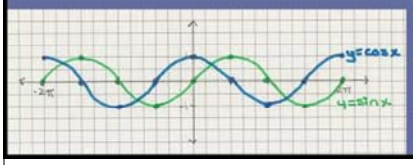
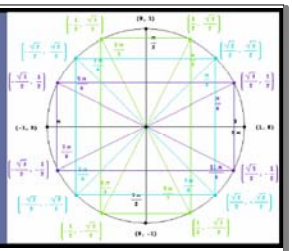
Unit 4: Trigonometry





4.6 Trigonometric Graphs

Terminology:

Periodic Function - a function that repeats itself after a certain amount of time

Period - time to complete one cycle

Sinusoidal Functions - wavy functions like Sine or Cosine

Median - middle of the graph

Amplitude - distance form the median to the max. or min. value

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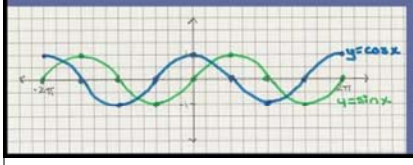
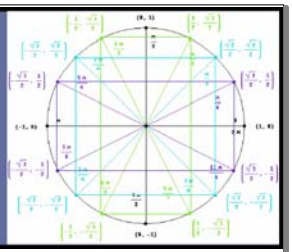
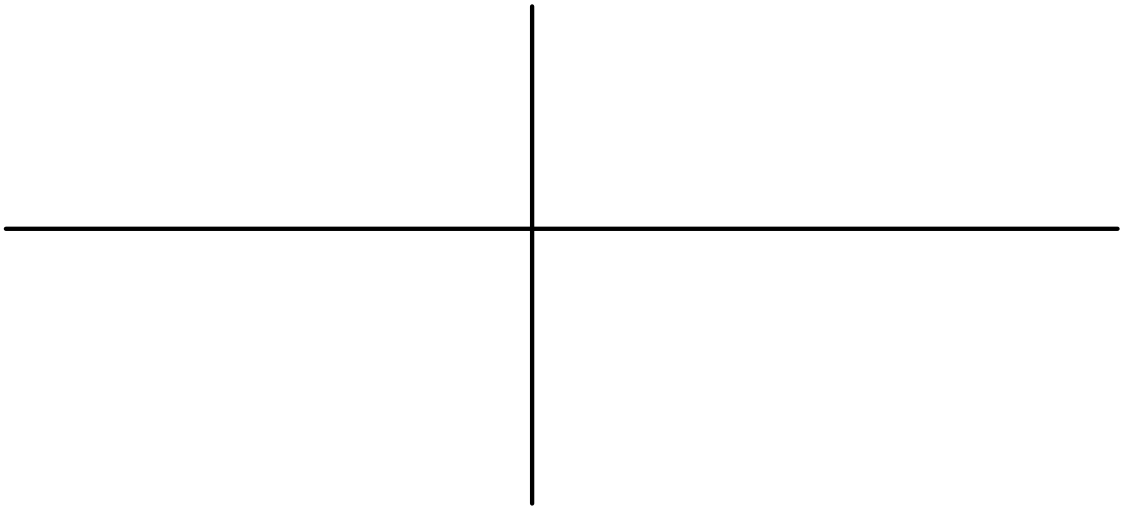




y = sin θ



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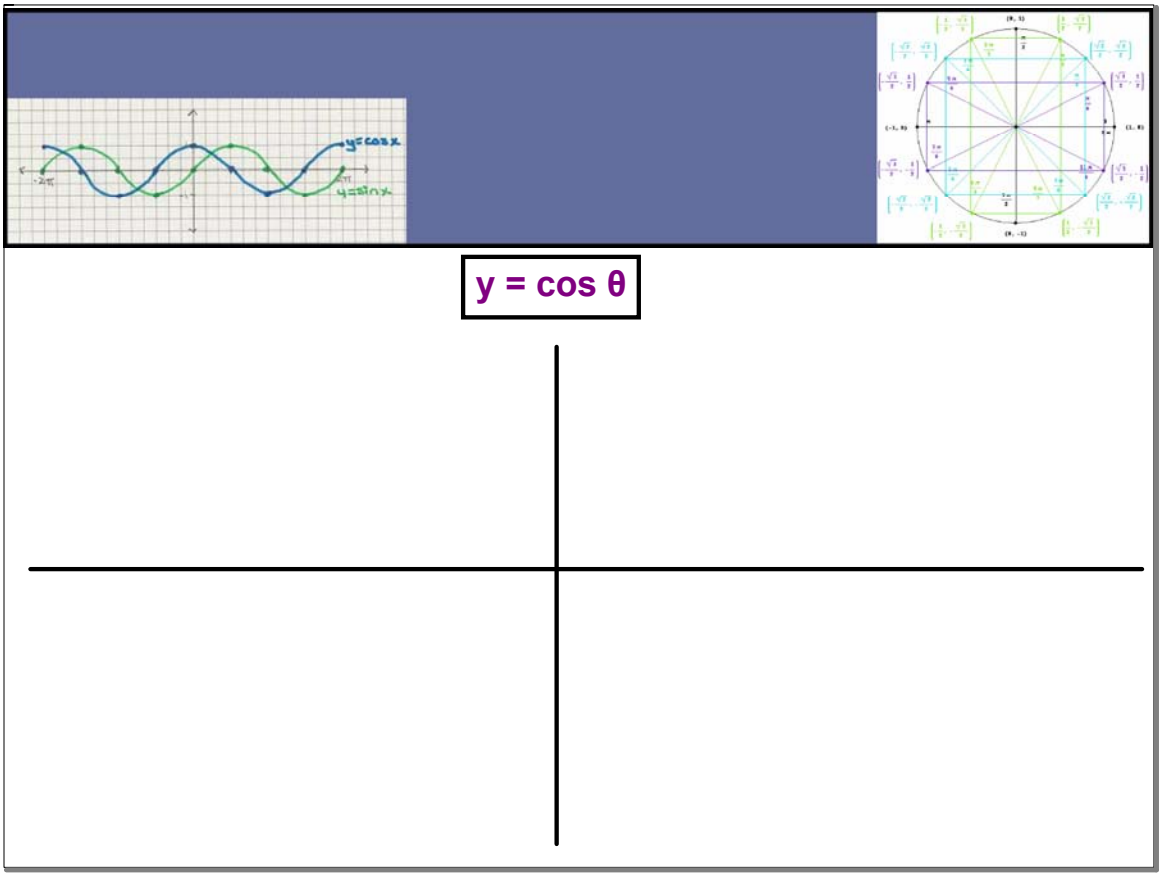

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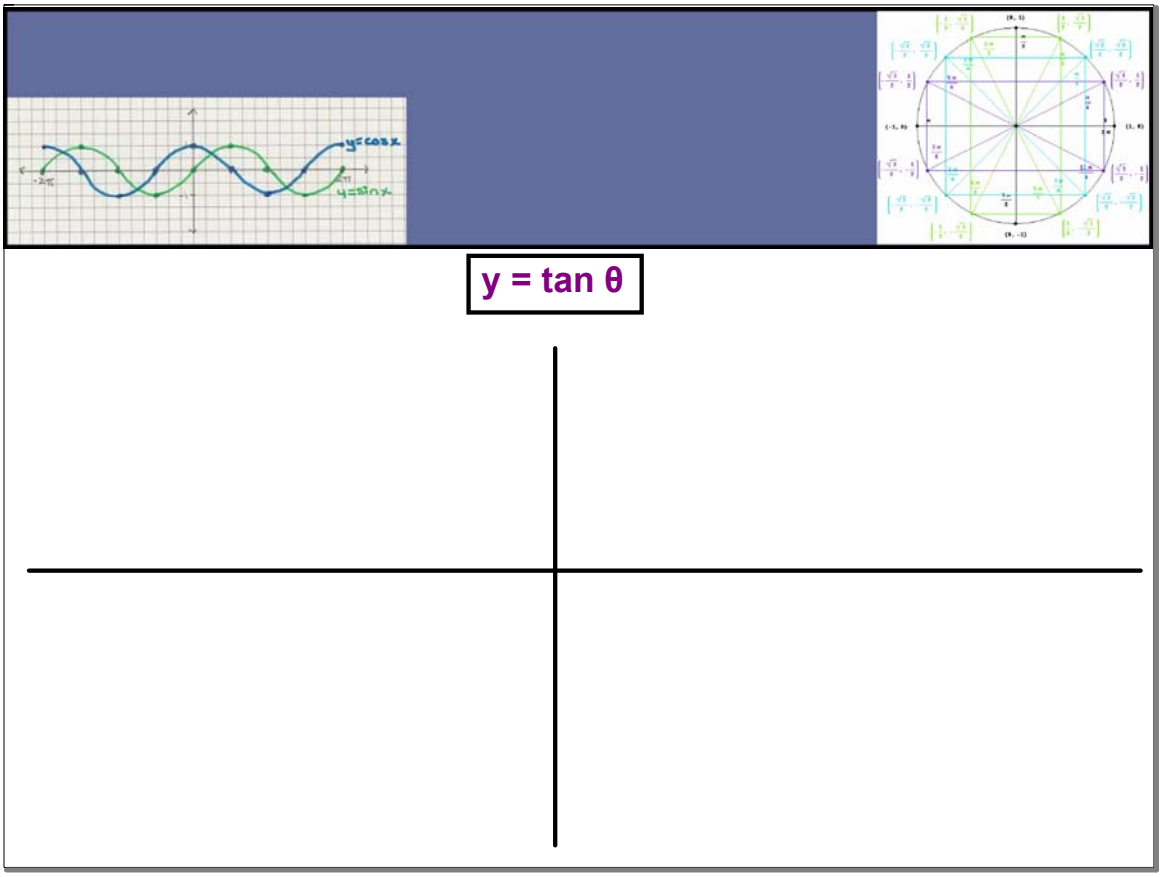

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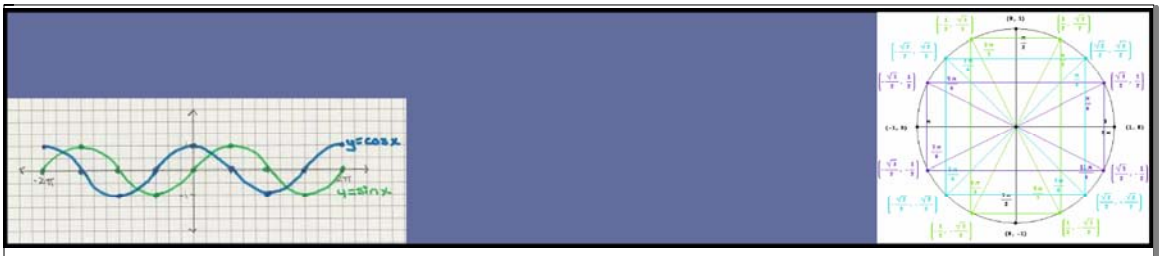
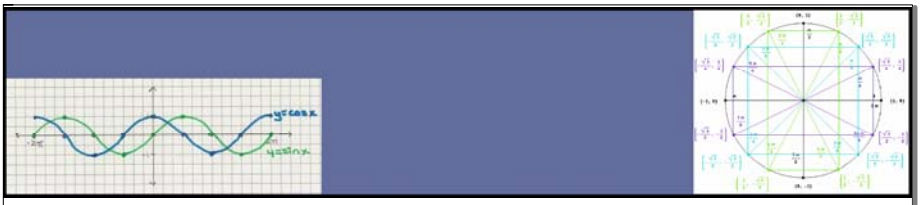


Transformations of Sinusoidal Graphs:

$$y = a\sin\left[b(x-c)\right] + d$$

$$y = a\cos\left[b(x-c)\right] + d$$

Nov 14-10:51 AM



Ex.) Compare the following and state the characteristics:

a) sin x and 2sin x

b) sin x and -3sin x

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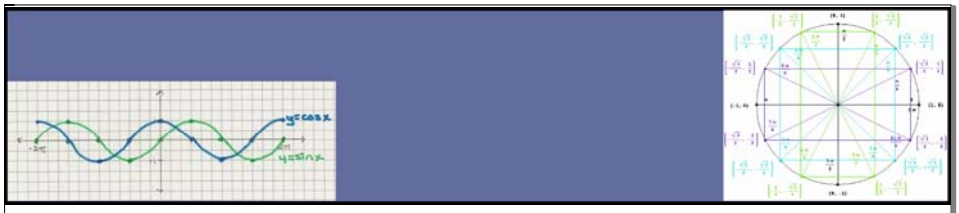
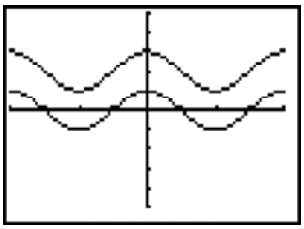
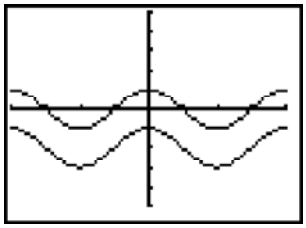


c) cos x and cos(x) + 2



d) cos x and cos(x) - 2



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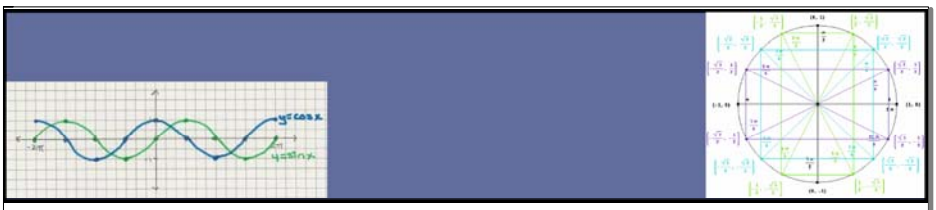
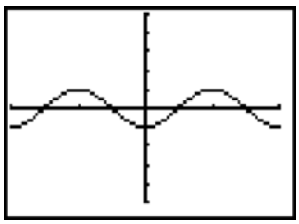
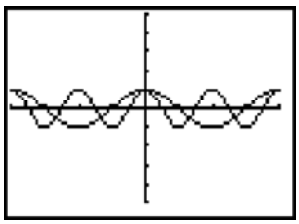


e) sin x and sin (x - 90°)



f) cos x and cos(2x)



Pg. 233 # 4-11, 13, 15.

Nov 14-10:56 AM

4.7 Writing Sinusoidal Equations

Determine a, b, c, d from the graph and sub into the equation on the formula sheet.

Recall: $y = a\sin[\, b(x-c)\,] + d$ & $y = a\cos[\, b(x-c)\,] + d$

a:

b:

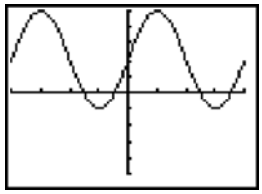
c:

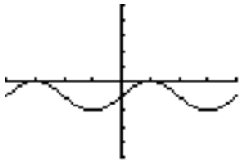
d:

Worksheet

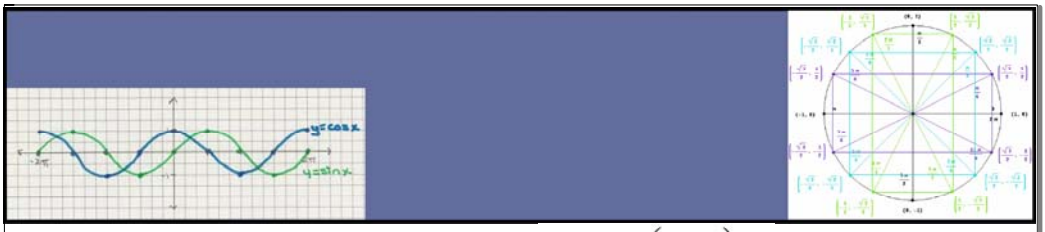
Nov 14-10:58 AM

Ex.) Determine the characteristics of the following sinusoidal functions.





Nov 14-1:01 PM



Ex.) A sine functions is given by the equation $y = 3\sin 2\left(x - \frac{\pi}{4}\right) + 2$. Determine:

a) Amplitude

b) Period

c) Phase shift

d) Median

e) Y-intercept

f) Domain

g) Range

h) Sketch

Dec 6-11:08 AM

Ex.) Consider the equation $y = 4\cos(x - \frac{\pi}{3}) + 2$. Determine the:

a) Amplitude

b) Period

c) Phase shift

d) Median

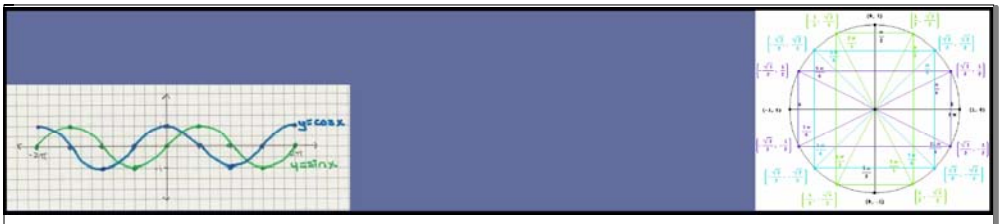
e) Y-intercept

f) Domain

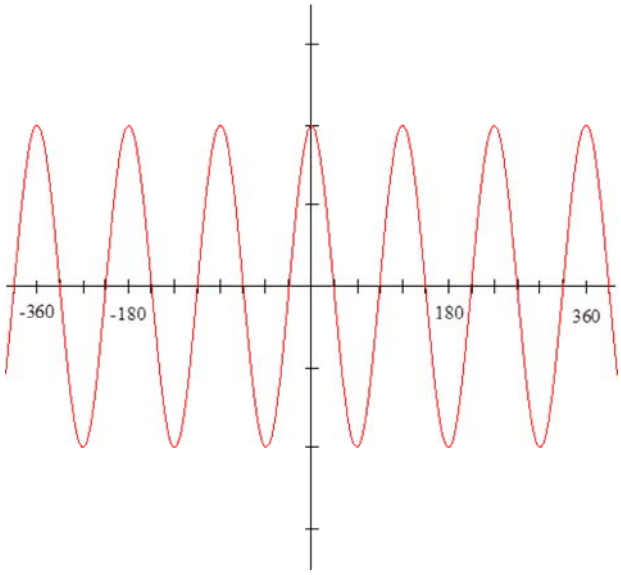
g) Range

h) Sketch

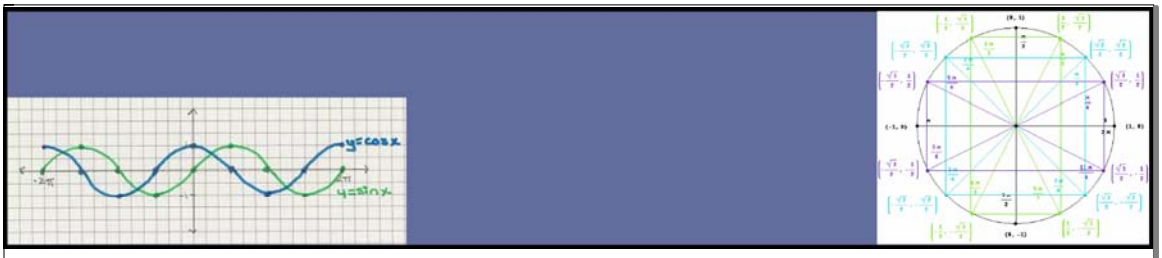
Dec 6-11:13 AM



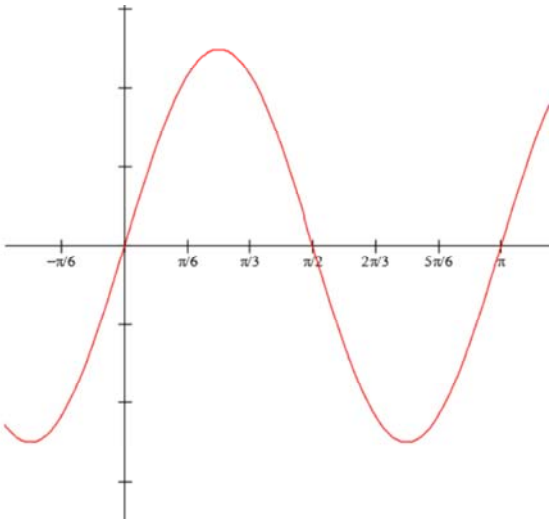
Ex.) What is the equation of the cosine function below?



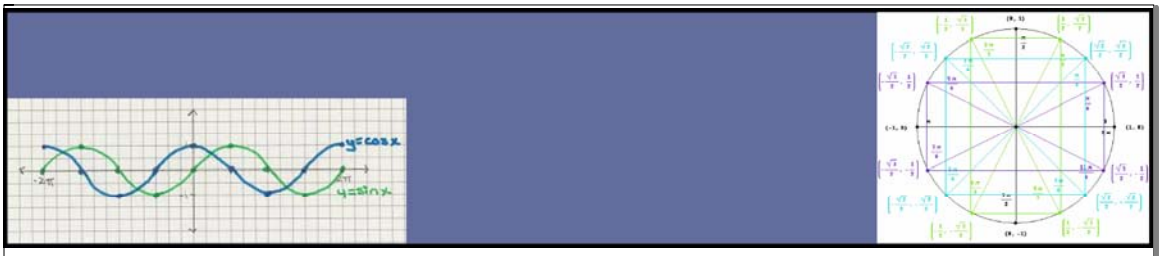
Dec 6-11:15 AM



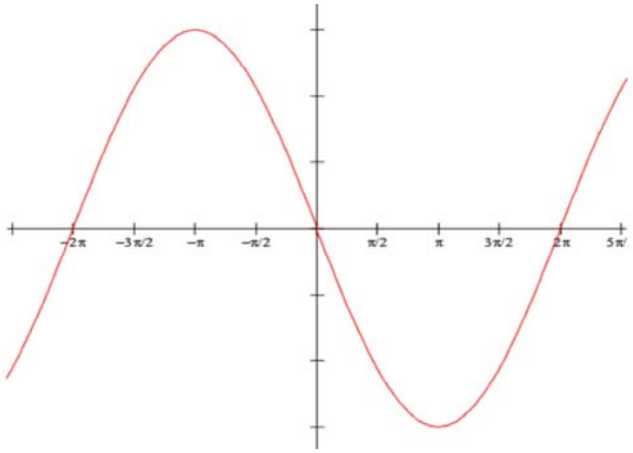
Ex.) What is the equation of the sine functions below?



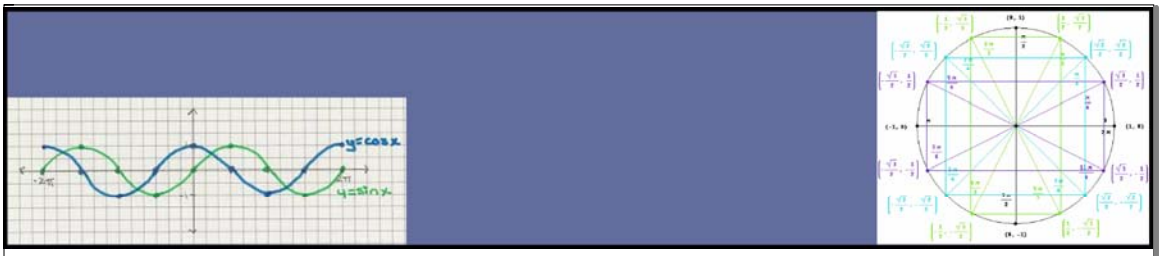
Dec 6-11:16 AM



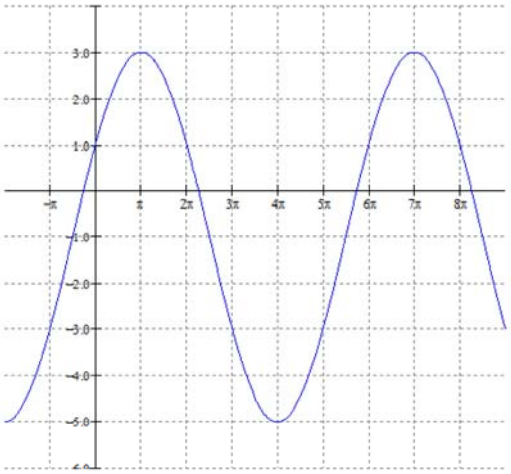
Ex.) What is the equation of the sine function below?



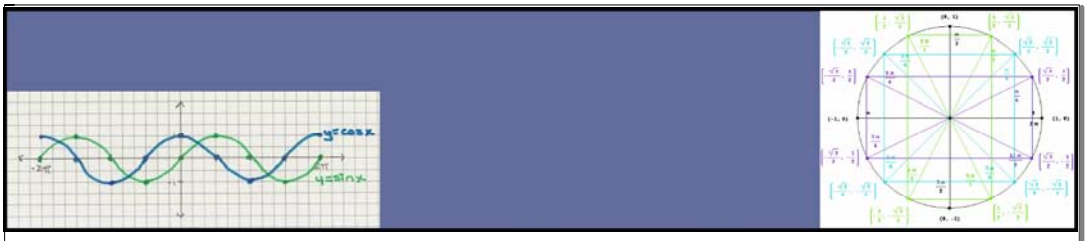
Dec 6-11:19 AM



Ex.) Determine the equation of the graph in the form $y = A\cos B(x - C) + D$.

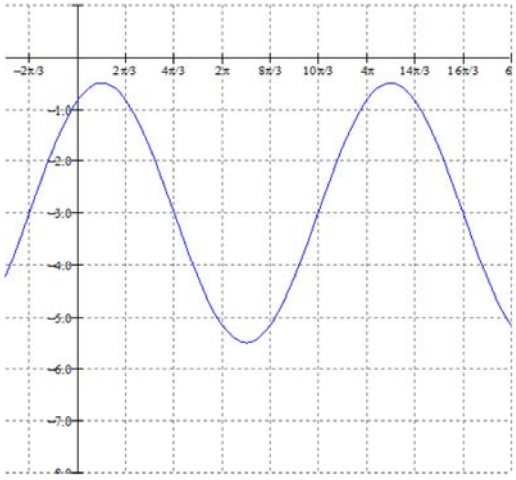


Dec 6-11:20 AM

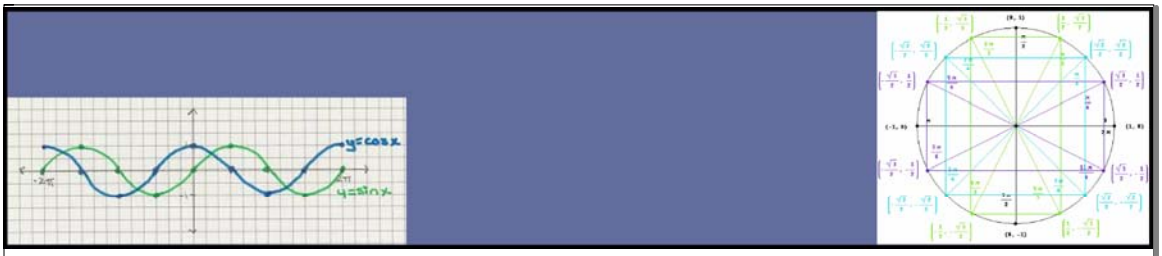


Ex.) Determine the equations of the graph in the following forms;

$$y = A\cos B(x-C)+D \qquad y = A\sin B(x-C)+D$$



Dec 6-11:22 AM



Trig Applications:

Ex.) By using the averages of high and low tide levels, the depth of water, *d(t),* in metres, in a seaport can be approximated by the sine function

$$d(t) = 2.5\sin 0.164\pi(t-1.5)+13.4$$

a) Graph the function on your calculator.

b) What is the period of the tide?

c) A cruise ship needs a depth of at least 12 m of water to dock safely. For how many hours per tide cycle can the ship dock safely?

Dec 4-8:06 AM

Ex.) A Ferris wheel has a radius of 20 m. It rotates once every 40 s. Passengers get on at a point S, which is 1 m above ground level. Suppose you get on at S and the wheel starts to rotate.

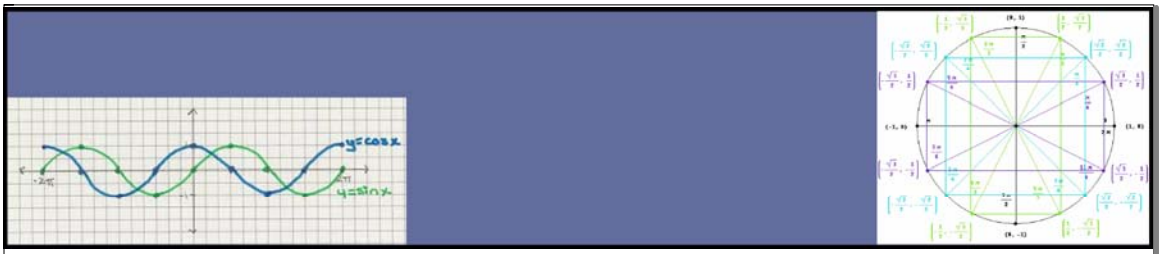
a) Graph how your height above the ground varies during the first two cycles.

b) Write an equation that expresses your height as a function of the elapsed time.

c) Estimate your height above the ground after 45 s.

d) Estimate one of the times when your height is 35 m above the ground.

Dec 4-8:10 AM



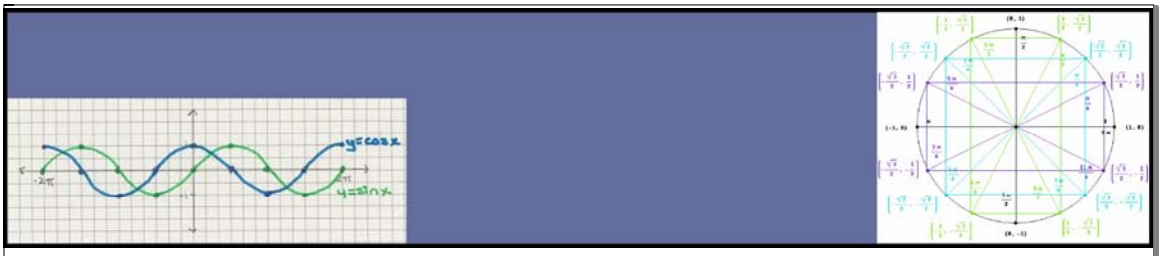
Ex.) A Ferris wheel has a diameter of 14 m. It makes 2 cycles in 60 s. If you get on the Ferris wheel at the lowest point, 1 m above the ground, determine

a) an equation that represents the motion of the Ferris wheel.

b) the height at 40 s.

c) the first time that you will reach a height of 15 m on the Ferris wheel.

Dec 4-8:14 AM



Ex.) According to statistical data from Environment Canada, the lowest normal daily temperature is -14˚C on January 26 and the greatest normal daily maximum temperature in Edmonton is 23˚C on July 26, day 208.

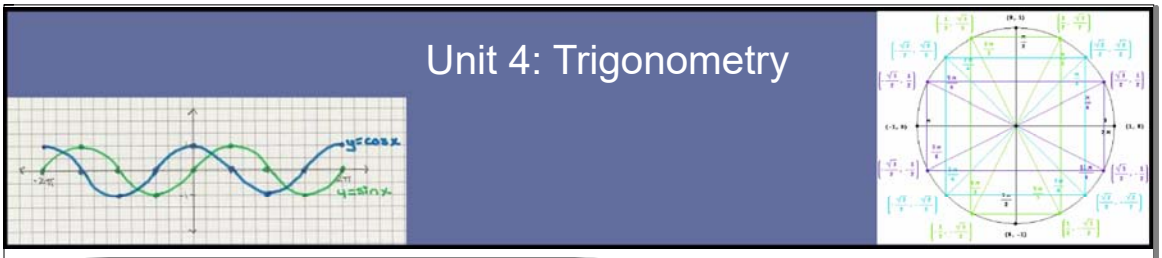
a) Write a sine and cosine function to approximate the temperature in Edmonton.

b) Use your equation to predict the maximum temperature on November 10, day 324.

c) What is the first day of the year where the maximum temperature is zero?

Assignment 3

Dec 4-8:16 AM



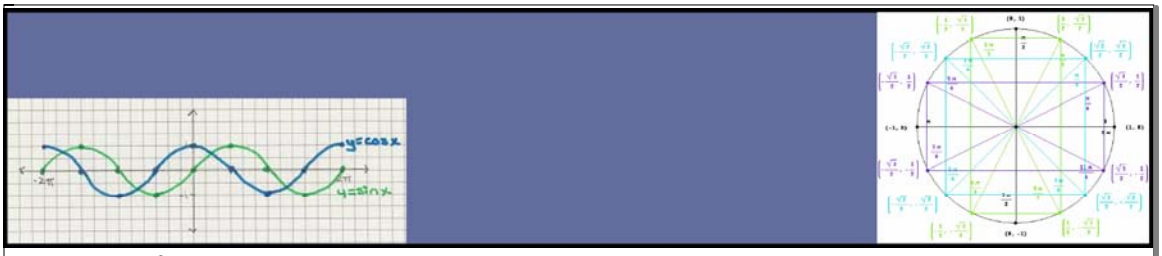
4.8 Solving Trig Equations with General Solutions

When giving a general solution for a trig equations, it means that there are infinite solutions as you can rotate around an infinite angle.

Ex.) For the following trig equations, give (a) the solution for [0^o, 360^o) and (b) the general solution.

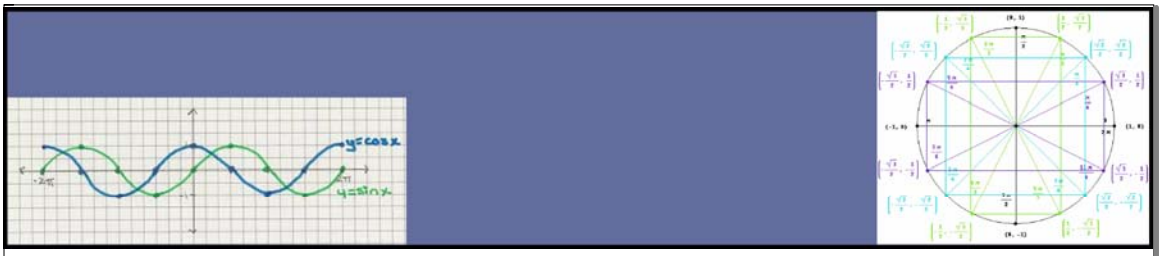
a) $2\cos\theta = \sqrt{2}$

Nov 14-12:57 PM



b) $2\cos^2 x - 1 = 0$

Nov 14-1:01 PM



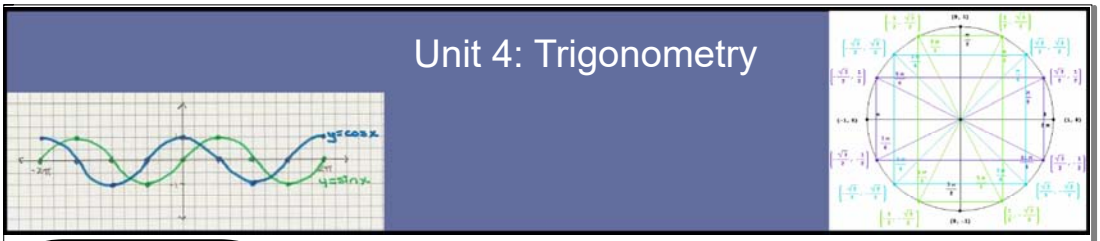
c) $16 = 6\cos[(\pi/6)x] + 14$

Nov 14-1:02 PM

d) $10 = 6\sin[(\pi/4)x] + 8$

Pg. 275 # 1, 4, 5, 6, 10.

Nov 14-1:03 PM



Unit 4: Trigonometry

4.9 Trig Identities

When working with Trig Identities, you can prove them three ways:

1) Graphically: LS = RS ($y_1 = y_2$) and see if the graphs overlap.

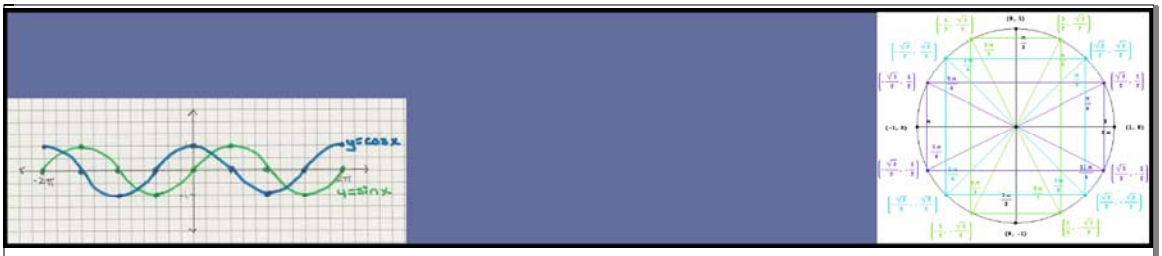
2) Numerically: Substitute an angle into the equations and check to see if LS = RS.

3) Algebraically: Simplify the identities using the formula sheet.

* This will be presented in a "Two-Column Proof"/"T-Form Proof" arrangement. **You never cross the equal sign.**

* Most problems can be solved by changing to sin and cosine.

Nov 14-1:05 PM

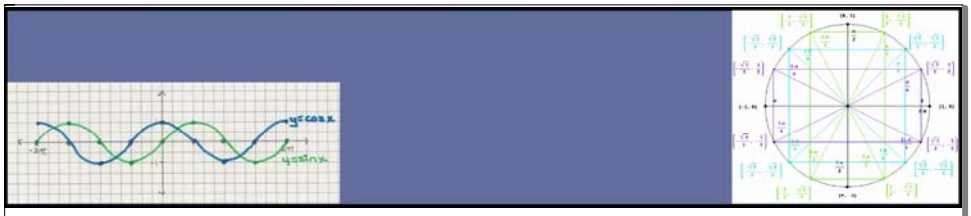


Common Misconception:

$\sin x^2$ vs. $\sin^2 x$

Helpful Understandings:

Nov 14-1:12 PM

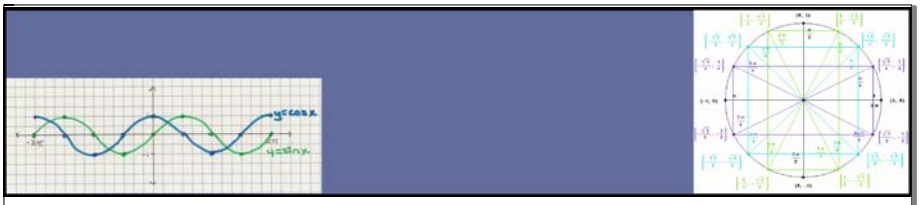


Ex.) Prove the following identities numerically:

a) $\sin\theta \cot\theta = \cos\theta$, when $\theta = 30°$

b) $\tan^2 x = \sec^2 x - 1$, when $x = \pi/7$

Nov 14-1:15 PM



Ex.) Prove the following identities algebraically:

a) $\frac{\sin x}{\tan x} = \cos x$, $\tan x \neq 0$

b) $\frac{1}{\cos x} - \cos x = \sin x \tan x$

Nov 14-1:18 PM

c) $\frac{\sec^2 x}{\sec^2 x - 1} = \csc^2 x$

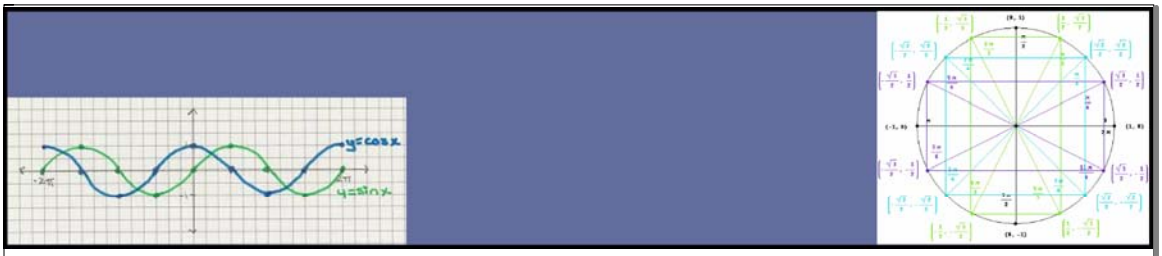
d) $\sin^3 x + \cos^2 x \sin x = \frac{1}{\csc x}$

Nov 14-1:21 PM

e) $(\tan x - 1)^2 = \sec^2 x - 2\tan x$

Worksheet

Nov 14-1:25 PM



f) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$

Nov 14-1:28 PM

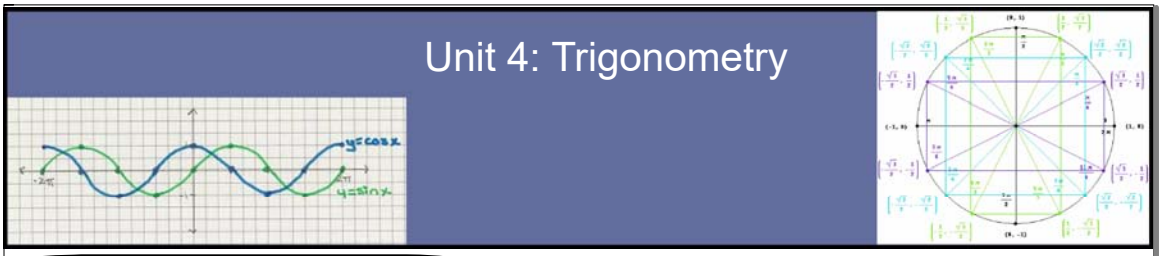
*Trick for fractions with binomial denominators...multiply by the conjugate.

g) $\frac{1 + \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 - \cos\theta}$

Finish worksheet.

Pg. 314 # 1-8.

Nov 14-1:29 PM



Unit 4: Trigonometry

4.10 Sum and Difference Identities

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

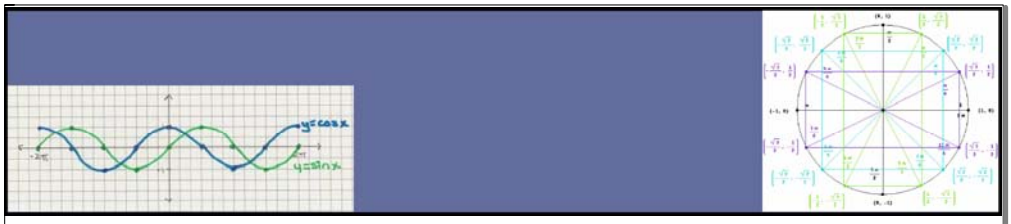
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Nov 14-1:32 PM

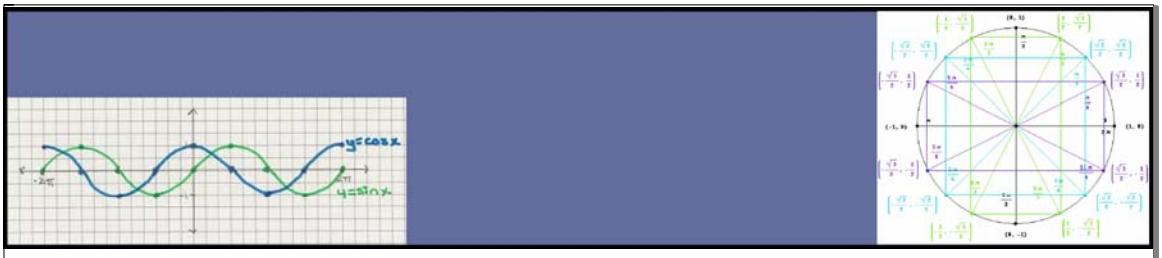


Ex.) Determine the exact value of angles that are not multiples of the standard reference angles (ie. 0, 30, 45, 60, 90, etc.).

a) cos (105°)

b) sin 15°

Nov 14-1:33 PM



c) tan 15°

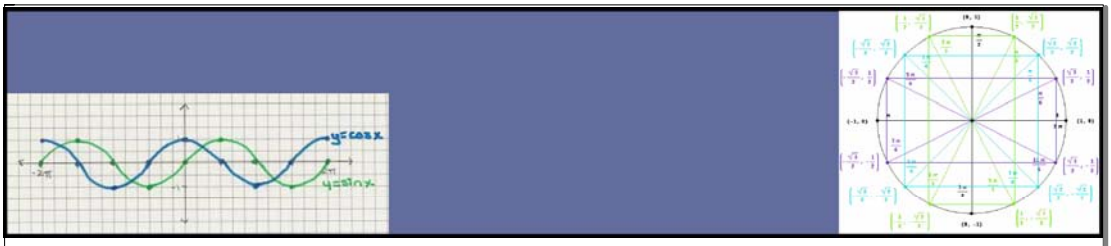
Nov 14-1:35 PM

Ex.) Simplify the following to a single trig function:

a) sin 35° cos 40° + cos 35° sin 40°

b) $\frac{\tan 10° + \tan 35°}{1 - \tan 10° \tan 35°}$

Nov 14-1:35 PM



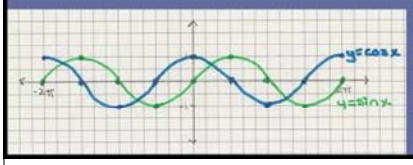
Ex.) Prove the following identity:

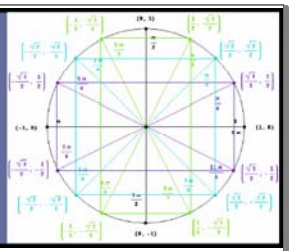
$$\sin(\pi/2 + M) = \cos M$$

Pg. 306 # 1abd, 2bd, 7, 8, 10, 11c, 17, 19, 20ab.

Nov 14-1:37 PM

Unit 4: Trigonometry





4.11 Double Angle Identities

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

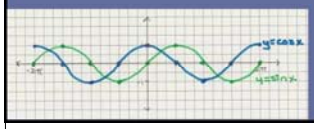
$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

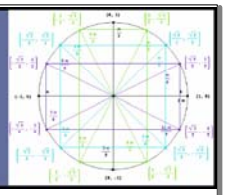
$$\cos(2\alpha) = 2\cos^2\alpha - 1$$

$$\cos(2\alpha) = 1 - 2\sin^2\alpha$$

$$\tan(2\alpha) = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

Nov 14-1:40 PM





Ex.) Simplify:

a)

b)

c)

Nov 14-1:41 PM

Ex.) Prove:

a)

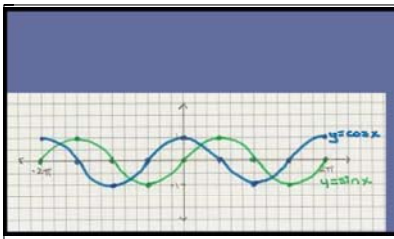
b)

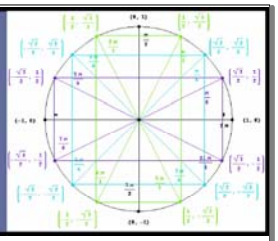
Nov 14-1:42 PM

c)

Ex.) Determine the exact value of cos (2x) when tan x = 5/12 and cos x < 0.

Nov 14-1:42 PM





Ex.) **Determine** the exact value of tan 2A when cos A = 3/5 in quadrant I.

Pg. 306 # 1c, 2c, 4, 5, 11ab, 20cd.

Nov 14-1:43 PM