

# Unit 4 Relations and Functions Notes.notebook

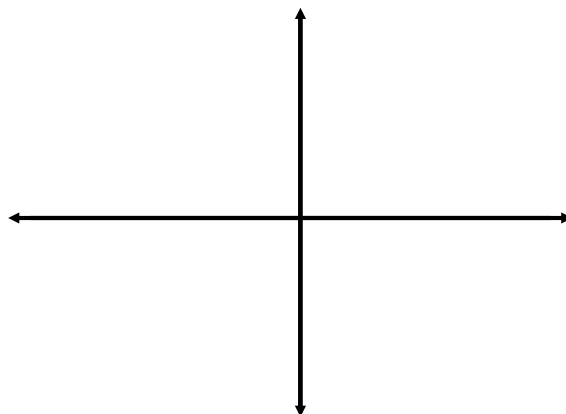
Functions

## Unit 4: Relations and Functions

Functions

$3 \rightarrow 3x - 4 \rightarrow 5$

## 4.1 Intro to Relations and Functions



Functions

## Unit 4: Relations and Functions

Functions

$3 \rightarrow 3x - 4 \rightarrow 5$

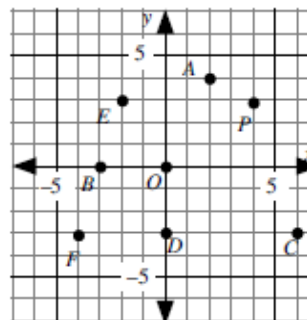
a) Complete the following by writing the coordinates of the points represented by the letters on the grid.

A(            )      B(            )      C(            )  
 D(            )      O(            )

b) Write the coordinates of the point in the second quadrant.

c) Write the coordinates of the point in quadrant III.

d) Complete the following table using "positive" or "negative".



| Quadrant | x-coordinate | y-coordinate |
|----------|--------------|--------------|
| I        |              |              |
| II       |              |              |
| III      |              |              |
| IV       |              |              |

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f(x)

## Functions

**Describing a Pictorial Pattern Using a Linear Relation**

Use the following information to answer this Class Example.

Three toothpicks are used to form a triangle. A second triangle is formed by adding two more toothpicks. A third triangle is formed by adding another two more toothpicks, and the pattern continues.

Example of a toothpick →

**Class Ex. #2**

a) Draw the next two diagrams in the pattern.

b) Complete the table relating the number of toothpicks  $P$ , to the number of triangles,  $T$ .

|                           |   |   |   |   |   |
|---------------------------|---|---|---|---|---|
| Number of Triangles, $T$  | 1 | 2 | 3 | 4 | 5 |
| Number of Toothpicks, $P$ | 3 |   |   |   |   |

c) Represent the data from the table of values on the grid.

d) Explain why it does not make sense to join the points in a straight line.

e) Describe in words the relationship between the number of toothpicks and the number of triangles.

f) Write an equation that can be used to determine the number of toothpicks if we know the number of triangles.

g) Use the equation to determine the number of toothpicks if there are 27 triangles.

h) Use the equation to determine the number of triangles if there are 83 toothpicks.

f(x)

## Functions

**Discrete and Continuous Variables**

In Class Example #3, the variables  $I$  and  $V$  are examples of **continuous variables** since they can take on every value within a particular interval, i.e. a variable for which it is possible to find an intermediate value between any two values. For the graph in this example, the current can take on any value between 0 and 5.

In Class Example #2, the variables  $P$  and  $T$  can only take on limited values (in this case whole number values) and are therefore NOT continuous variables. Such variables are called **discrete variables**.

A graph relating two discrete variables consists of a series of unconnected points, whereas in the graph of two continuous variables the points would be connected.

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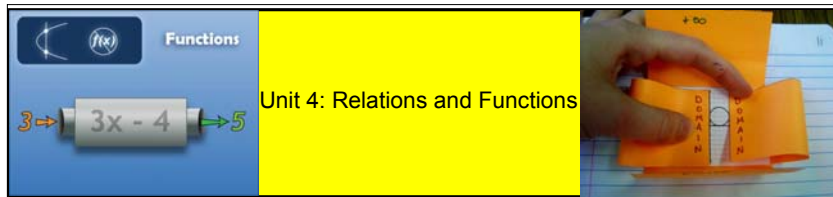
**Class Ex. #4**

Classify each of the following variables as discrete or continuous.

a) time taken to complete a 100 m sprint    b) number of students who pass Math 10

c) height of students    d) shoe size

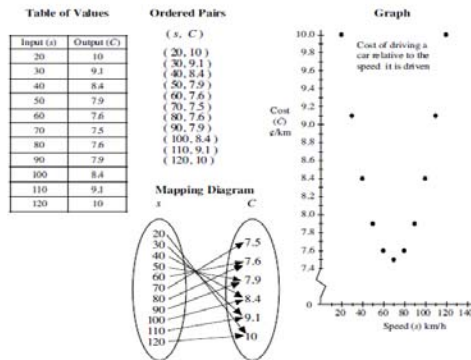
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## 4.2 Relationships Between Variables

A comparison between two sets of elements is called a **relation**. This can be represented in the following ways:

- ~ Words
- ~ Table of Values
- ~ Mapping Diagram
- ~ Equation
- ~ Ordered Pairs
- ~ Graph
- ~ Function Notation



$$C = 0.001s^2 - 0.14s + 12.4$$



## Terminology

- Dependent Variable - variable along y-axis
- Independent Variable - variable along x-axis
- Input - x
- Output - y
- Domain - under which x-values does the graph live (all possible x's)
- Range - under which y-values does the graph live (all possible y's)

$$y = 3x - 5$$

- ~ dependent variable
- ~ outputs of the relation
- ~ 2nd coordinate of an ordered pair
- ~ graphed on the vertical axis

- ~ independent variable
- ~ inputs of the relation
- ~ 1st coordinate of an ordered pair
- ~ graphed on the horizontal axis

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3 →

3x - 4

→ 5

Consider the relation described by the equation  $y = 2x - 5$ .

a) Complete the first five rows of the following table of values which shows some of the possible input values.

| Input<br>(x) | Output<br>(y) | Ordered pair<br>(x,y) |
|--------------|---------------|-----------------------|
| -2           |               |                       |
| -1           |               |                       |
| 0            |               |                       |
| 1            |               |                       |
| 2            |               |                       |
|              |               |                       |
|              |               |                       |

b) Show the data in a mapping diagram.

Functions

3 →

3x - 4

→ 5

b) Plot the ordered pairs in a) on the grid provided.

c) Connect the points on the grid, and extend the line in both directions with arrows at both ends.

d) Use the graph to determine the value of  $y$  when  $x = 6$ .

e) Use the equation to determine the value of  $y$  when  $x = 6$ , and verify the answer in d).

f) Write the value of  $y$  when  $x = 6$  in the table of values using the first blank space in a).

g) Use the graph to determine the value of  $x$  when  $y = 3$ . Put this information in the last row in a).

h) Complete the following statement:

This relation is called a \_\_\_\_\_ relation because the graph of the relation is a straight line.

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Functions

## Unit 4: Relations and Functions

### 4.3 Domain and Range

**\*\*Big, Important Idea\*\***

The **domain** of a relation is the set of all possible values which can be used for the **input** of the **independent variable (x)**.

The **range** of a relation is the set of all possible values of the **output** of the **dependent variable (y)**.

#### Discrete Data

Rule:

Domain: {all x-values}

Range: {all y-values}

#### Continuous Data

Rule:

Domain:  $\{x | a \leq x \leq b, x \in \mathbb{R}\}$

Range:  $\{y | c \leq y \leq d, y \in \mathbb{R}\}$

*\*put the smallest value first and the biggest value second*

Functions

## Unit 4: Relations and Functions

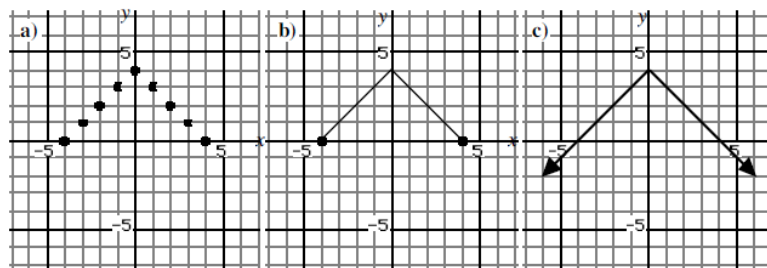


List the domain and range of the following set of ordered pairs.

- a) (1, 2), (0, 5), (3, 8), (5, 9), (-3, 2)      b) (3, 3), (0, 3), (-3, 3), (2, 9), (-8, 3)




In each case, state the domain and range of the relation represented by the graph.



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⤵
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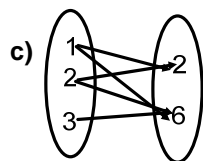
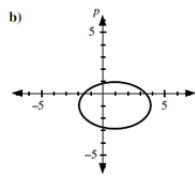
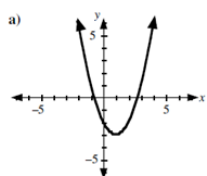


3 →

$3x - 4$

→ 5

Ex.) State the domain and range for each relation:



d) A circle with center  $(-2, 3)$  and a radius of 3.

⤵
⦿
Functions



3 →

$3x - 4$

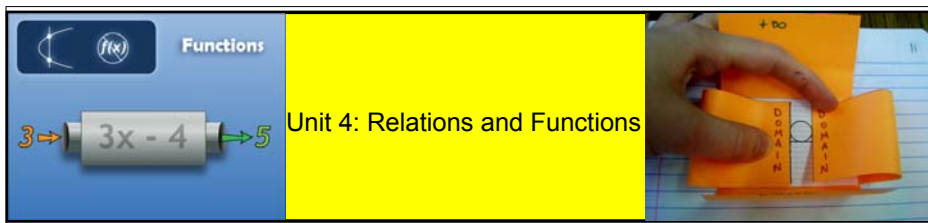
→ 5

Ex.) Draw the graph of a relation which has a domain of  $x \in \mathbb{R}$ , range  $\{y | y \leq 2, y \in \mathbb{R}\}$  and:

- i) only one x-intercept                      ii) two x-intercepts

b) Can you draw a graph which has domain of  $x \in \mathbb{R}$ , range  $\{y | y \leq 2, y \in \mathbb{R}\}$  and no x-intercepts?

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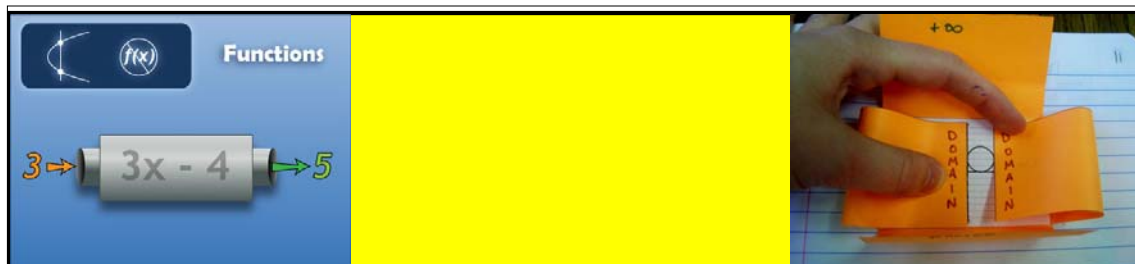
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4.4 Graphing Relations

[Graph the relation  \$y=2x-6\$](#)

- Sketch the graph.
- Write an appropriate window.
- Determine the value of  $y$  when  $x = 10$ .
- Determine the value of  $x$  when  $y = 8.75$ .
- State the  $x$  and  $y$  intercept.
- Determine the domain and range.

\*Won't be tested on this until Unit 5



Steps to Graph:

| TI-NSpire                                   | TI-84 (Plus)    |
|---|-----------------|
| 1. B: Graph.                                | 1. $y=$ button. |
| 2. MENU--3: Graph Entry--1: Function        | 2. $y_1=2x-6$   |
| 3. $f1(x)=2x-6$ (this just means $y=2x-6$ ) | 3. GRAPH        |
| 4. ENTER                                    |                 |

\*to find things on your graph everything is under MENU-- Analyze Graph.

\*to find things on your graph everything is under 2nd Trace



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4.5 Analyzing Graphs

"Lisa purchases a new car for \$20 000. The value of the car can be represented by the formula  $V = 20\,000 - 1250t$ , where  $V$  is the value of the car in dollars and  $t$  is the age of the car in years."

- a) • Graph  $V = 20\,000 - 1250t$  using a graphing calculator.
  - Use the ZoomFit feature as a guide to adjust the graphing window. Then use the Window key to align the graph appropriately for this scenario. Write the window setting below.
  - Sketch the graph on the grid provided.
- b) Calculate the  $t$ -intercept of the graph using a graphing calculator, and label it on the sketch. Describe what this value represents in the context of the question.
- c) Calculate the  $V$ -intercept of the graph using a graphing calculator, and label it on the sketch. Describe what this value represents in the context of the question.
- d) Use the trace feature of a graphing calculator to determine what the car will be worth in 5 years.
- e) Use the intersect feature of a graphing calculator to determine when the car will be worth half of the purchase price. Illustrate this on your sketch.
- f) Write an appropriate domain and range for the function which describes the value of the car over time.

Functions

Unit 4: Relations and Functions

Class Ex. #2

The height of a human cannon ball, "Cano", can be described by the formula  $h = 12 + 6t - t^2$ , where  $h$  is the height in metres above ground level and  $t$  is the time in seconds. Cano is projected out of a cannon from the top of a building and lands on a soft mat. The mat is placed in a hole in the ground so that the top of the mat is level with the ground.

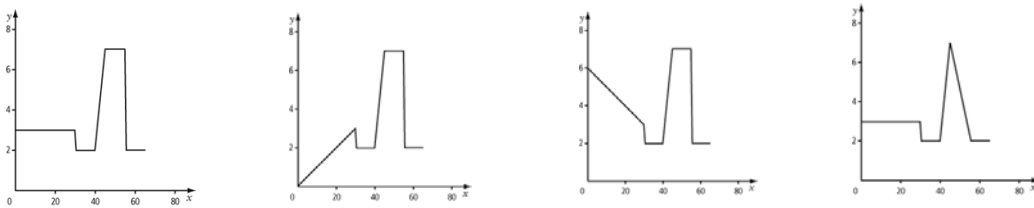
- a) Display the graph of  $h = 12 + 6t - t^2$  on a graphing calculator.
- b) Write down a window setting which would be appropriate for this situation.
- c) Sketch  $h = 12 + 6t - t^2$  on the grid provided.
- d) What is the height of the cannon above the ground?
- e) What is the maximum height Cano reaches?
- f) How many seconds does it take Cano to reach the highest point on the path he is travelling?
- g) To the nearest hundredth of a second, how long does it take Cano to land on the mat?
- h) How high is Cano one second after he is launched?
- i) When will Cano be at the height in h) again?
- j) In words, describe the relation connecting height and time.
- k) Write an appropriate domain and range for the relation described in j).



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Ex.) A runner goes for a jog. She jogs at a constant speed for 30 minutes, then walks at a slower speed for 10 minutes. She then starts jogging and increases her speed over a 5 minute period until she reaches her maximum speed. She keeps up that pace for 10 minutes. Finally, she walks at a constant speed for 10 minutes until she returns to her starting point. Which of the following graphs illustrates this situation?



Functions

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## 4.6 Functions


A [function](#) is a special type of relation in which each element of the domain has only one element in the range.

Vertical Line Test:

If every vertical line drawn intersects the graph **exactly once**, then the relation **is a function**.

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$f(x)$ 
Functions



$3 \rightarrow$ 

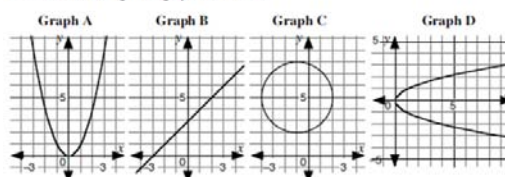
$3x - 4$

 $\rightarrow 5$

How we can determine which relations are functions by...

a) a graph

Each of the following is the graph of a relation.



b) a mapping diagram

c) ordered pairs

$f(x)$ 
Functions



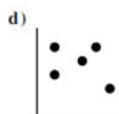
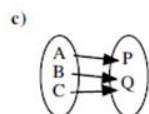
$3 \rightarrow$ 

$3x - 4$

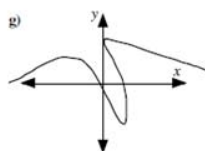
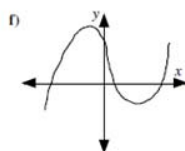
 $\rightarrow 5$

Determine which of the following are functions. Explain your answers.

- a) (5, 8), (6, 7), (-5, 3), (2, 3), (6, 8)      b) (3, 3), (2, 3), (4, 5), (-3, 2)



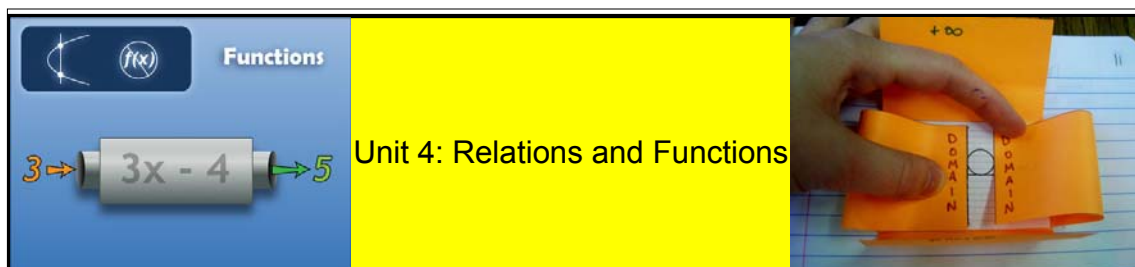
e) The relation connecting the provinces and territories of Canada with their capital cities.



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Ex.) In the function  $y = 2x - 5$ , the domain is  $\{-1, 0, 1\}$ . What is the range?



### 4.7 Function Notation

Function notation is another way of writing an equation. Function notation allows us to specify what x value we are inputting into our equation.

**Main Idea:  $f(x)$  replaces  $y$**

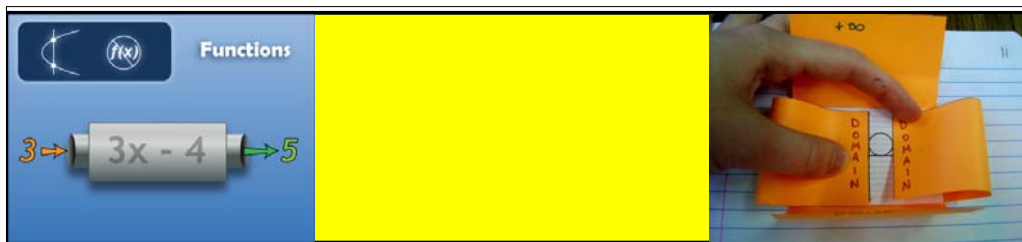
Function Notation

$$\begin{aligned}f(x) &= 2x + 3 \\f(5) &= 2(5) + 3 \\f(5) &= 13\end{aligned}$$

Equation of a Graph of the Function

$$\begin{aligned}y &= 2x + 3 \text{ when } x=5 \\y &= 2(5) + 3 \\y &= 13\end{aligned}$$

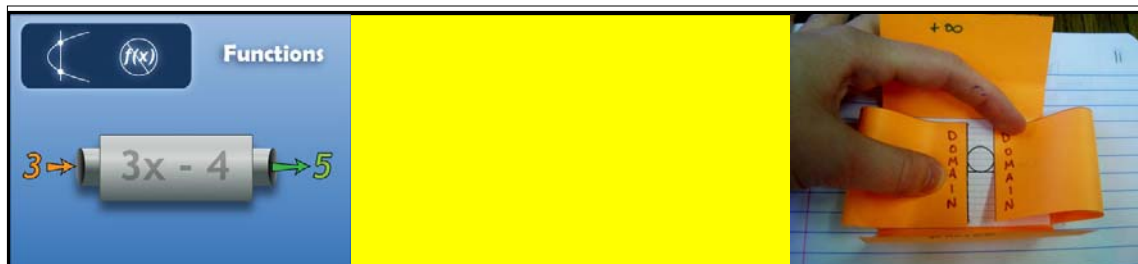
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Ex.) Consider the function  $f(x) = 4x + 5$  and  $g(x) = 4 - x$ .  
Evaluate:

- a)  $g(1)$                       b)  $f(-2)$                       c)  $g(-2)$                       d)  $f(0) - g(0)$

Ex.) If  $P(x) = -6x + 1$ , determine a simplified expression for  $P(2x)$ .



Ex 3) Consider the function  $f(x) = 8x - 5$ .

- a) Evaluate  $f(4)$                       b) Solve the equation  $f(x) = 11$

c) Solve the equation  $f(x) = 75$ .

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### Important Skill:

You need to be able to tell the difference between:

$$f(x) = 5 \quad \text{and} \quad f(8)$$